



Signal Flow & Radiometer Equation

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Antenna Basics

The **HartRAO 26m telescope** => **equatorially mounted Cassegrain** radio telescope

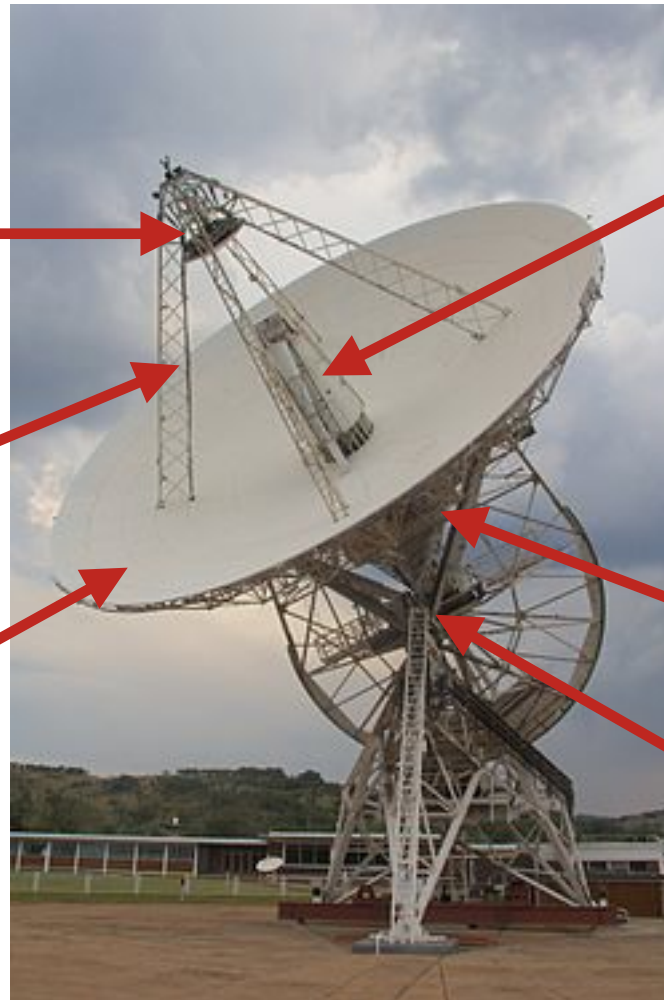
The **antenna reflectors** concentrate incoming E-M radiation into the focal point of the antenna

Secondary reflector

Sub-reflector (small reflector of hyperbolic curvature in front of the focus of the main reflector).

Sub-reflector support legs

Primary reflector



Feed housing (feed horns receivers and support structure)



Converts E-M radiation in free space to electrical currents in a conductor.

26 m telescope receivers (7):

1.6, 2.3, 5, 6.7, 8.4, 12.2 GHz
5 & 8.4 GHz **dual beam**
new 22 GHz cooled receiver
15 GHz **dual beam** coming

Deck Room

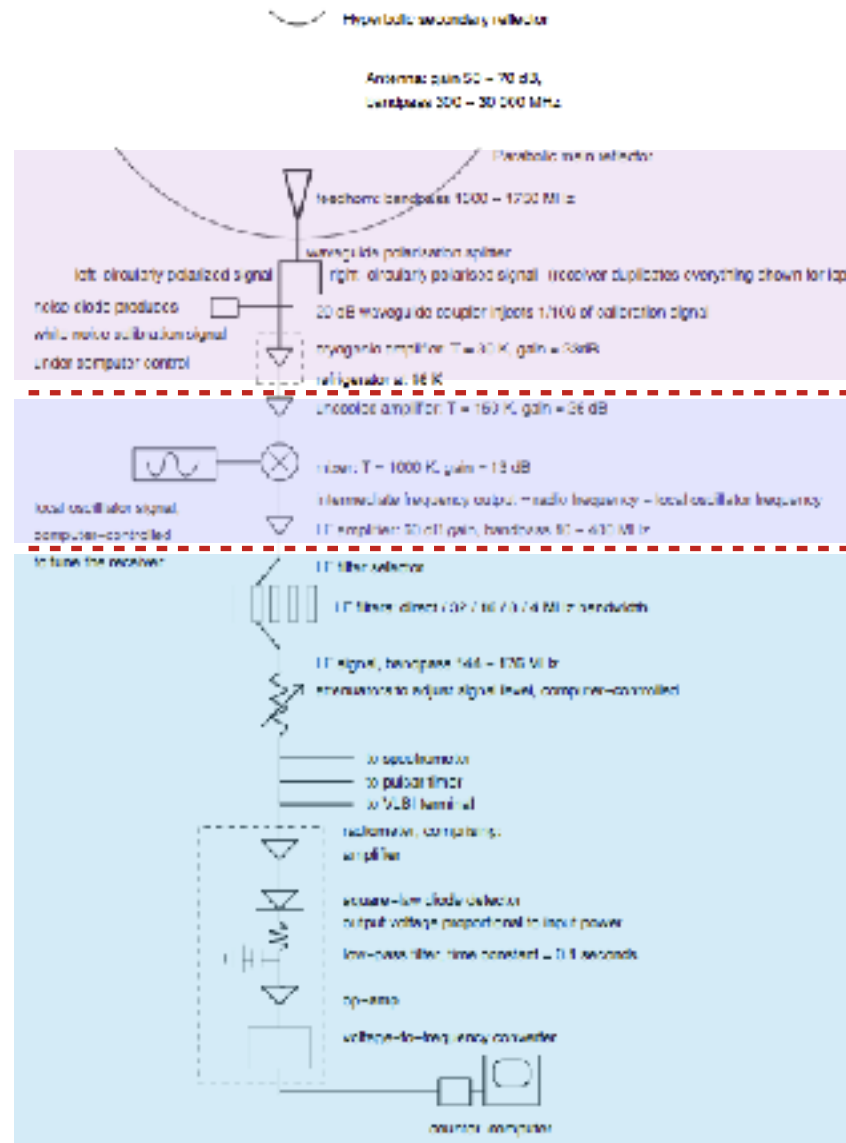
Local oscillator and mixers

Antenna positioner

The **antenna positioner** points the antenna at the desired location in the sky.

Antenna Basics

Signal chain: Main components of a typical **microwave receiver** and **radiometer**



Feed housing

Deck Room

Control Room

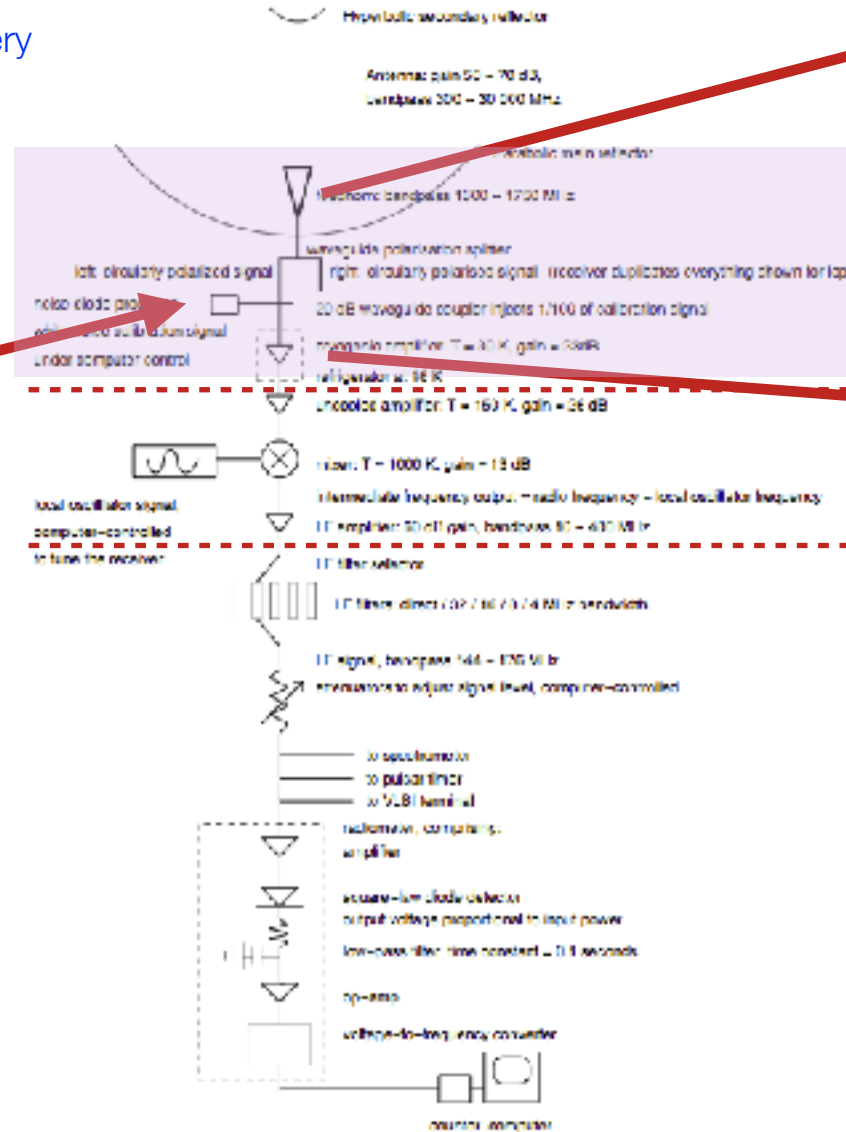
Antenna Basics

Signal chain: Main components of a typical **microwave receiver** and **radiometer**

Incoming signal: are very faint and noise like.

Feed horn and waveguide (to connect feed horn to first amplifier). All incoming signals are split into **LCP & RCP** by a hybrid waveguide polarisation splitter feeding LCP to one receiver chain and RCP to the other.

To calibrate the system a high stability **noise diode** injects a known noise signal which is split equally by a power divider between the LCP and RCP receiver chains.



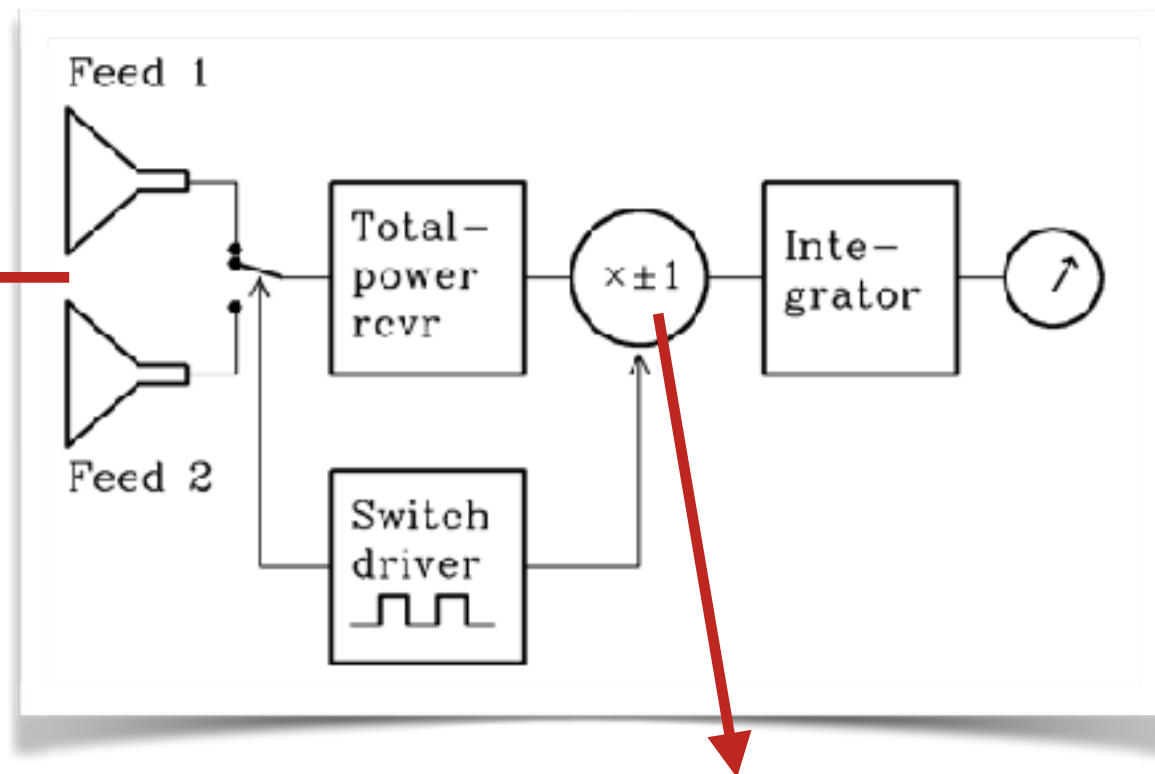
Amplification to a detectable level through a **low-noise amplifier**. Because the internal noise in the amplifiers is generally much larger than the signal, specially designed amplifiers that are **cryogenically cooled** are used to maximize sensitivity.

Antenna Basics

Signal chain: Main components of a typical **microwave receiver** and **radiometer**

If **feed 1** is pointing at the source (angular size of source smaller than separation of the beams from the two feeds) then **feed 2** will point off-source but measure nearly the same sample of atmosphere in the near field.

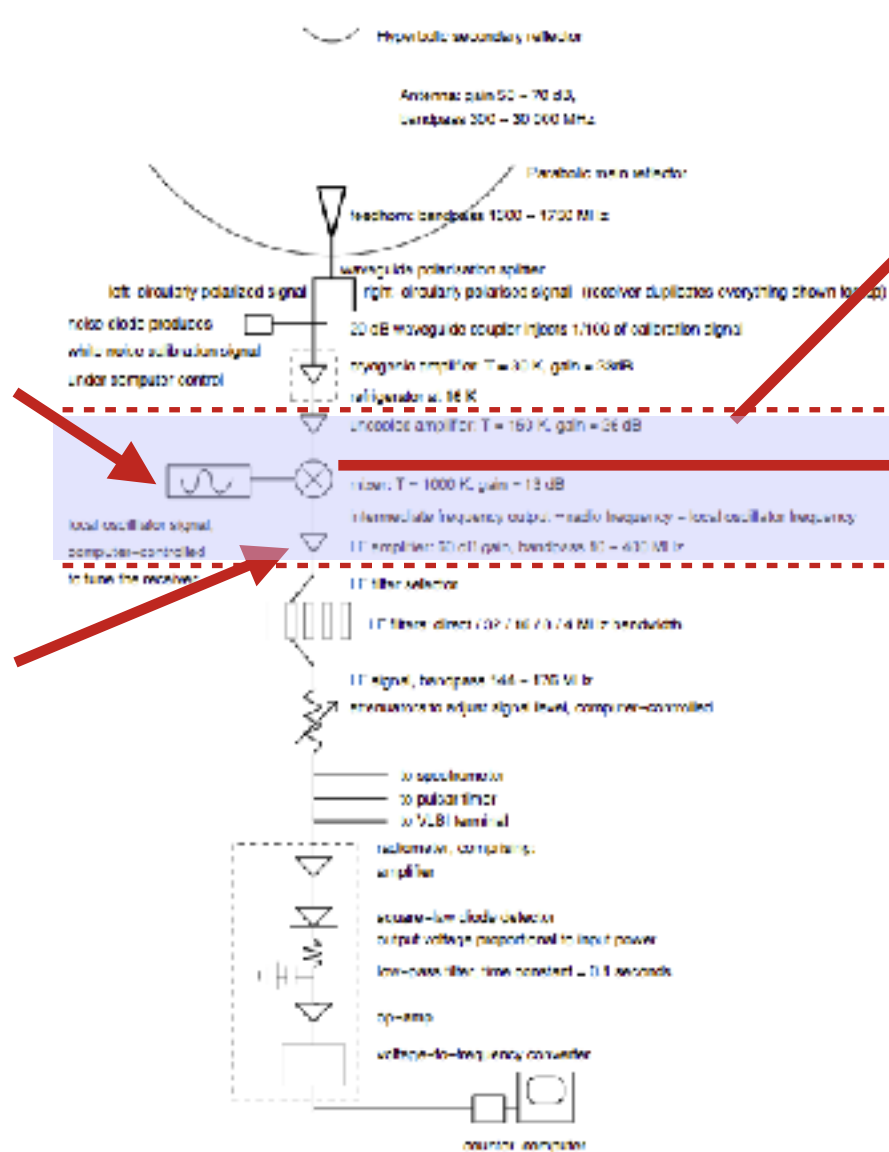
Dicke-switching: switching rapidly between two **identical feed horns** that are installed **East-West** next to each other on the telescope.



Output of receiver is **multiplied by +1** when receiver is connected to **feed 1** and by **-1** when connected to **feed 2**. Fluctuations in atmospheric emission and drifts due to changes in receiver gain are canceled for frequencies below the switching rate.

Antenna Basics

Signal chain: Main components of a typical **microwave receiver** and **radiometer**



RF signal is **down converted** to a lower frequency in order to minimise signal losses in coaxial cable).

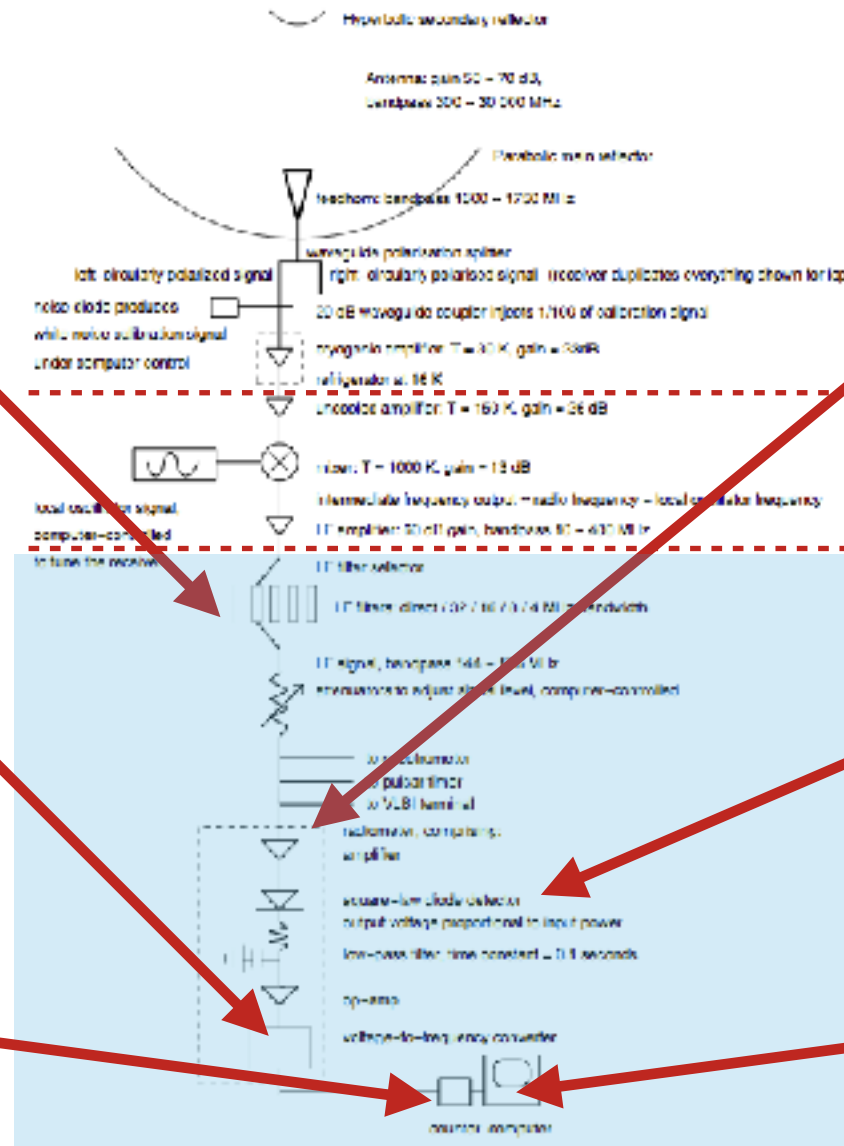
Local oscillator signal: computer-controlled to tune the receiver

To get the final output the IF signal is amplified, this time using an **IF Amplifier**

The **mixer** multiplies the RF signal with the **local oscillator signal**. The output signal that is used is the difference frequency component (RF - LO) of the product and is called the **intermediate frequency (IF)**.

Antenna Basics

Signal chain: Main components of a typical **microwave receiver** and **radiometer**



IF signal can be used unfiltered, or passed through **4, 8, 16 or 32-MHz bandwidth filters** to exclude interference from external signals at some observing frequencies.

The **radiometer** is the basic instrument for measuring the power of the incoming signal. The simplest form of radiometer is the **“total power”** type shown

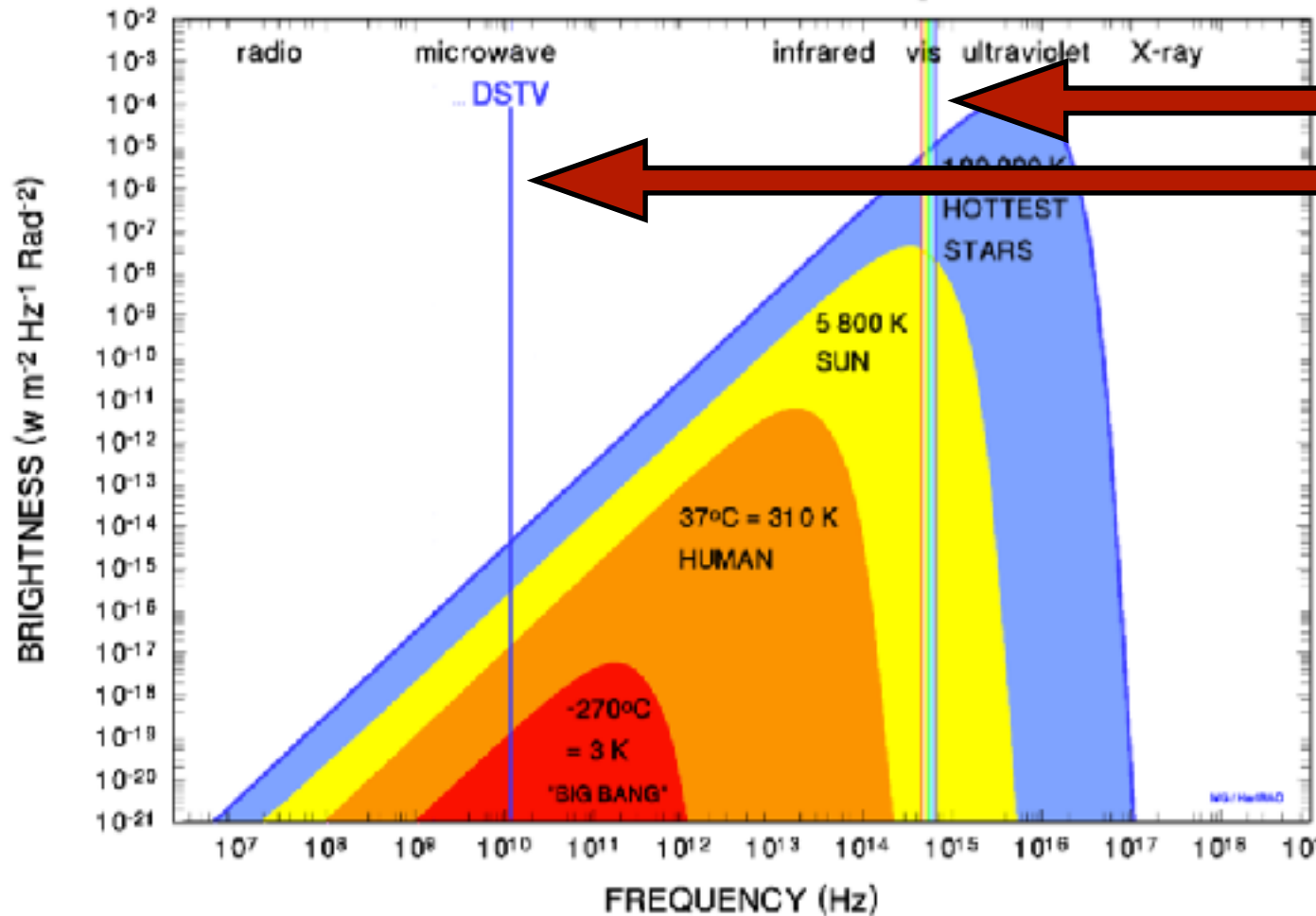
Voltage to frequency converter converts the signal to a square wave train (amplitude remains constant but the frequency is proportional to the DC voltage input).

The signal is then detected by a **Square law detector** which converts the IF signal into an output DC voltage proportional to the input power.

These oscillations are then measured with a **counter** such that the count rate (in units of Hertz) is proportional to the original IF signal's power.

Signals are loaded onto the Hart26m server in **FITS (Flexible Image Transport System)** format

Theory: T_B and T_A



For a black body radiator, the Brightness B is given by;

$$B = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

[W m⁻² Hz⁻¹ sr⁻¹]

Rayleigh-Jeans Law:

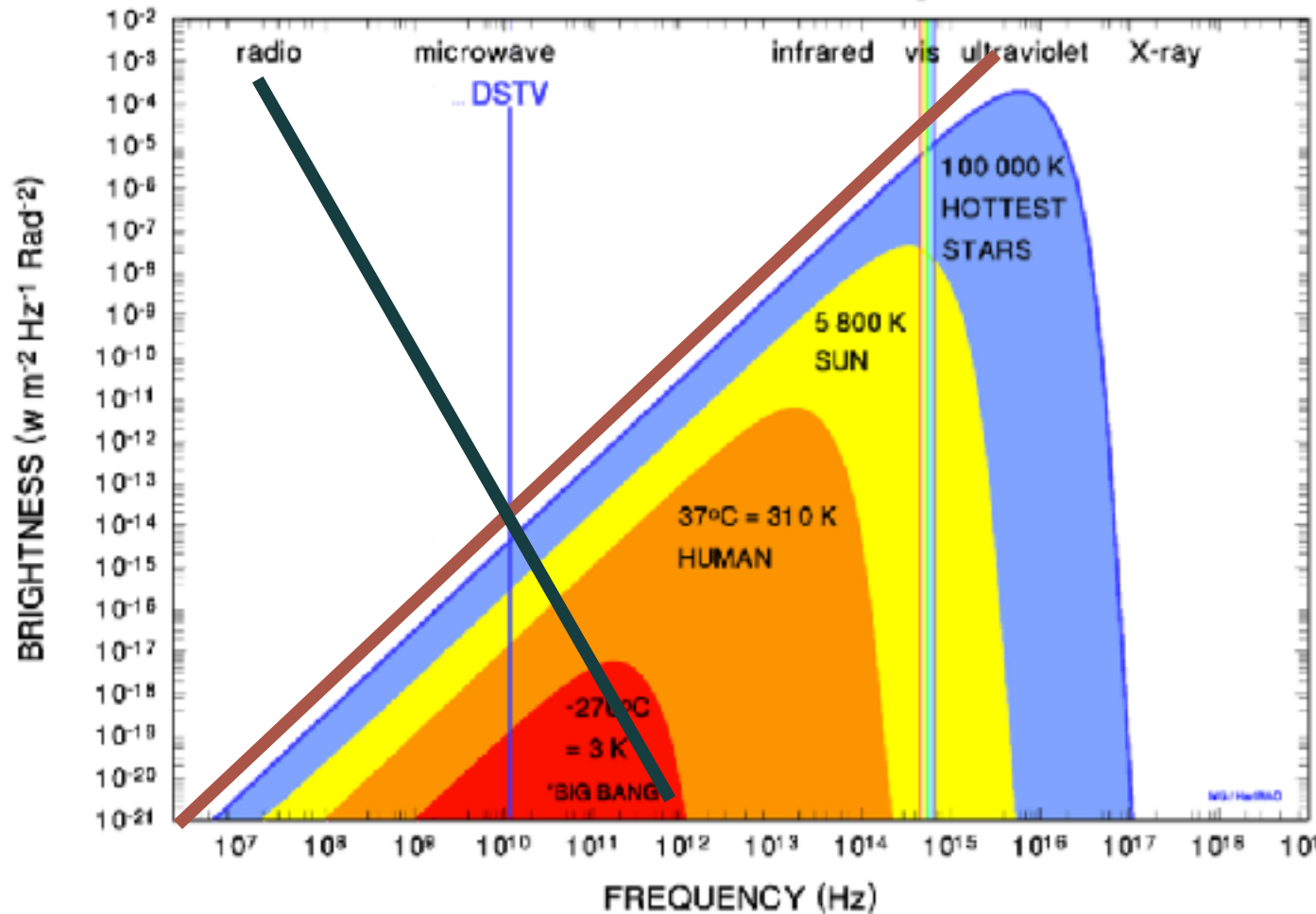
The brightness B and hence the power measured by a radio telescope is proportional to the temperature T of the emitting source

$$h\nu \ll kT, \quad B = \frac{2kT}{\lambda^2}$$

Blackbody radiation from solid objects of the same angular size, at different temperatures.

Brightness as a function of frequency.

Theory: T_B and T_A



$$h\nu \ll kT, \quad B = \frac{2kT}{\lambda^2}$$

$$[\text{W m}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1}]$$

Rayleigh-Jeans Law holds all the way through the radio regime for any reasonable temperature.

In the **Rayleigh-Jeans limit** a black body has a temperature given as;

$$T_B = B\lambda^2 / 2k \quad [\text{K}]$$

For some astronomical objects T_B measured by a radio telescope is meaningful as a physical temperature.

Radiation mechanisms are often **non-thermal** => effective temperature that a black body would need to have.

- **“Blank” sky** ~ 2.73 K (thermal big bang BB radiation)
- **Sun** at 300 MHz = 500000 K (mostly non-thermal)
- **Orion Nebula** at 300 GHz ~ 10-100 K (“warm” thermal molecular clouds)
- **Quasars** at 5 GHz ~ 10^{12} K (non-thermal synchrotron)

Theory: Detecting Radio Emission

- When the telescope looks at a radio source in the sky, the receiver output is the sum of radio waves received from **several different sources**:

The sum of these parts is called the **system temperature**

Sky temperature $T_{\text{sky}} \sim 10 \text{ K}$

$$T_{\text{sys}} = T_{Bcmb} + T_A + T_{at} + T_{wv} + T_g + T_R \text{ [K]}$$

CMB radiation coming from every direction in space. $\sim 2.7 \text{ K}$ at 1.4 or 4 GHz, reducing to 2.5 K at 12 GHz (but at lower frequencies the radio emission from the Milky Way becomes increasingly stronger.)

The emission from the **radio source** we want to measure, which produces the antenna temperature.

Radiation from the **dry atmosphere**. Adds about 1 K.

Radiation from the **water vapour** in the atmosphere. At 12 GHz adds 1 - 2 K, depending on the humidity.

The amplifiers in the antenna produce their own electronic noise, **receiver noise temperature**.

The radiation the feed receives through the antenna sidelobes from the (warm $\sim 290 \text{ K}$) **ground**. Adds 5 - 15 K pointing straight up at zenith, and increases when pointing close to the horizon.

Detecting Radio Emission from Space

- The antenna needs to be **calibrated to convert the signal amplitude in units of Hertz to units of Antenna Temperature in Kelvins [K]**, as it is the standard physically meaningful scale used with most radio analysis techniques.
- The output signal from the radiometer is proportional to the T_{sys} , from which we can extract the T_A .

$$T_{sys} = T_{Bcmb} + T_A + T_{at} + T_{wv} + T_g + T_R \text{ [K]}$$

- Prior to each drift scan, the **noise diode injects a noise signal with a known temperature** and this is used to **calibrate the antenna**.
- Comparing the noise diode's temperature to its count rate - can derive a conversion factor [K/Hz] to convert from counts (Hz) to antenna temp (K).

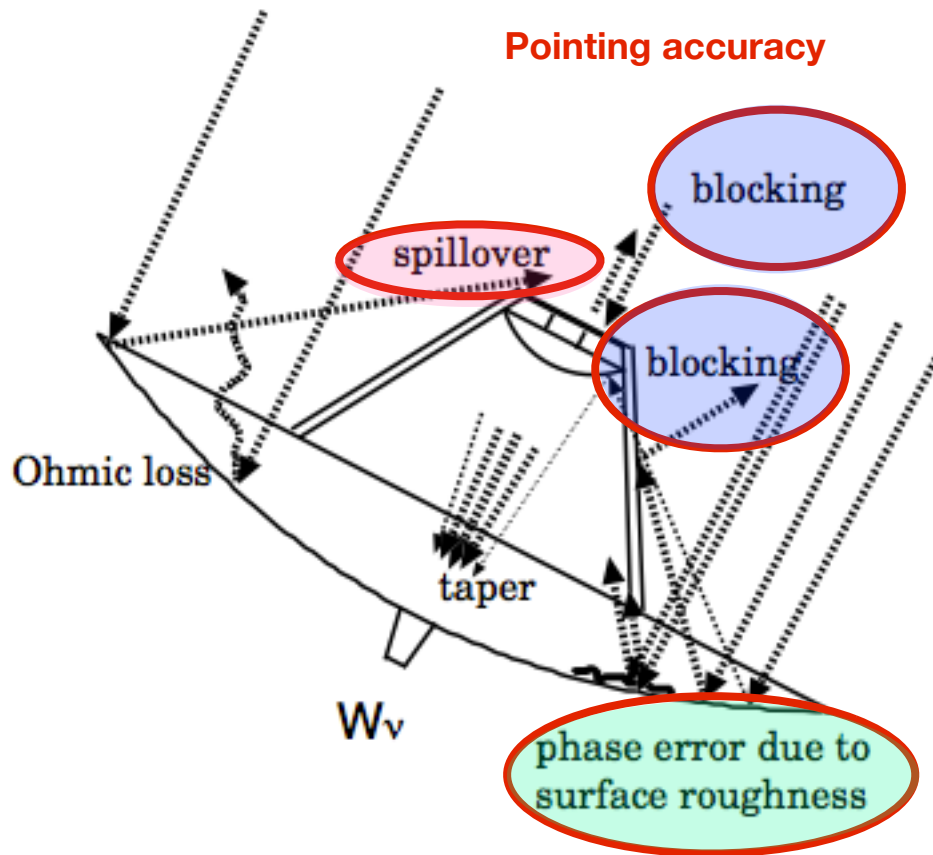
Theory: T_B and T_A

- The “**antenna temperature**” T_A of a source is the increase in in temperature (receiver output) measured when the antenna is pointed at a radio emitting source.
- NB: The **antenna temperature** has nothing to do with the physical temperature of the antenna.
- The **antenna temperature** will be less than the **brightness temperature** if the source does **not fill the whole beam** of the telescope. Must also correct for the **aperture efficiency**.

$$T_B = \frac{\Omega_A T_A}{\Omega_s \epsilon_m} \text{ [K]}$$

- By pointing the antenna at objects of known temperature that completely fill the beam we can calibrate the output signal in units of absolute temperature (Kelvins). One can think of a radio telescope as a remote-sensing thermometer.

Theory: Radio Telescope Antennas



As the radio emitter moves away from the middle of the beam the angle of the waves hitting the beam changes.

When all waves from each part of dish are in phase => strongest signal.

Moving away from the centre => destructive interference

Telescope sensitivity falls to a minimum => phase difference of about 1λ across diameter of dish

Factors reducing the aperture efficiency (0.80, 0.75, 0.64)

Radiation Basics

- The source flux density S , is the product of the brightness and source solid angle

$$h\nu \ll kT, \quad B = \frac{2kT}{\lambda^2} \quad [\text{W m}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1}]$$

$$S = \frac{2kT\Omega_s}{\lambda^2} \quad [\text{W m}^{-2} \text{ Hz}^{-2}]$$

Remember !!! $1 \text{ Jy} = 10^{-26} [\text{W m}^{-2} \text{ Hz}^{-2}]$

Radiation Basics



- It is important to note that the **flux density** of a radio source is **intrinsic** to it, and the same flux density should be measured by any properly calibrated telescope. However the antenna temperatures measured for the same emitter by different telescopes will be proportional to their effective collecting areas.
- We can now calibrate the telescope at each frequency of interest. We can carry out scans of **standard calibrator sources** (Ott et al. 1994) and measure the peak antenna temperature in each polarisation.

Radiation Basics



- For convenience, we often refer to the **Point Source Sensitivity (PSS)**, which is the number of Kelvins of antenna temperature per polarisation, obtained per Jansky of source flux density. This is also known as the **‘DPFU’** or **‘Degrees per Flux Unit’**.
- For the HartRAO 26 m telescope the *PSS* is typically about 5 Jy/Kelvin per polarisation. The **PSS** in each polarisation is simple to determine experimentally from the measured T_A of calibrator sources of known flux density. **NB: unpolarised sources => half the total flux density is received in each polarisation.**

$$PSS_{lcp} = \frac{(S/2)}{K_s T_{Alcp}} \text{ and } PSS_{rcp} = \frac{(S/2)}{K_s T_{Arcp}} \text{ [Jy K}^{-1} \text{ per polarisation]}$$

- Theoretically the values for the two polarisations should be the same; in practise there is always a small difference between them, and data from each polarisation should be corrected using the value appropriate for that polarisation.

Radiometer Equation

- So you have a telescope - with certain characteristics
 - .. and some given observations - with certain characteristics
 - ... some kind of weather, hardware working a certain way ...
- The question is: Can you see the source you want to see ?
- The end result - **RADIOMETER EQUATION** all about **Signal to Noise**

$$\frac{S}{N} = \frac{T}{T_{sys}} \sqrt{\Delta\nu\tau}$$

why do astronomers use all these temperatures ?

Radiometer Equation

- Radio Astronomers like to think of their telescopes as resistors
- .. and when you put power into a resistor
- ... it heats up

$$h\nu \ll kT, \quad B = \frac{2kT}{\lambda^2} \quad [W m^{-2} Hz^{-2} Sr^{-2}]$$

Rayleigh-Jeans Law holds all the way through the radio regime for any reasonable temperature.

- The question is: what flux density is received by your antenna ?

$$\int B d\Omega = S \quad [W m^{-2} Hz^{-2}]$$

Remember !!! $1 \text{ Jy} = 10^{-26} [W m^{-2} Hz^{-2}]$

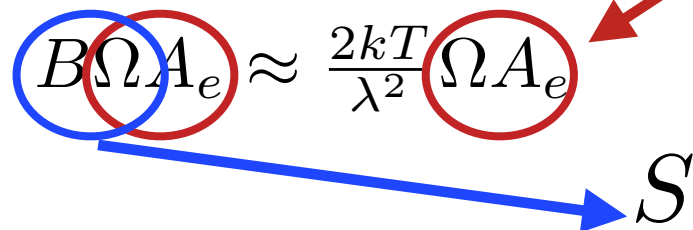
Radiometer Equation

- Now lets look at the power that we actually received by the antenna at a given frequency

... we integrate the flux density over the area of the antenna

$$\int S dA = P \text{ [W Hz}^{-2}\text{]}$$

- Now the antenna theorem states: $A_e \Omega = \lambda^2$
- Lets go one step back from power (without using fancy integration)
... what we effectively just did was ...

$$B \Omega A_e \approx \frac{2kT}{\lambda^2} \Omega A_e$$


$$S A_e = 2kT$$

Radiometer Equation

- We have now converted successfully between flux density and source temperature

$$T = \left(\frac{A_e}{2k} \right) S$$

- This quantity is known as the “forward gain” of the antenna ... property of a given antenna -> k/Jy or Jy/k

Radiometer Equation

- Now lets talk about T_{sys} ...

$$\frac{S}{N} = \frac{T}{T_{sys}} \sqrt{\Delta\nu\tau}$$

$$T_{sys} = T_{sky} + T_R$$

- T_{sky} - everything above your antenna you don't want to detect - depends on frequency
- T_R - thermal noise of the electrical components in your receiver (mixers / amplifiers - anything with charge carriers that “jitters” around at a given temperature (-> cool components))

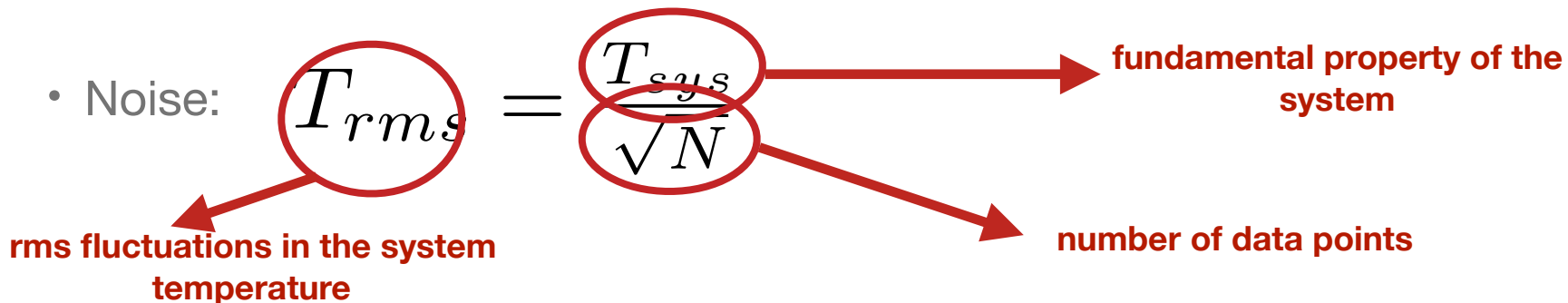
Radiometer Equation

- Typically $\rightarrow \frac{S}{N} = \frac{T}{T_{sys}} \sqrt{\Delta\nu\tau}$

$$T < T_{sys}$$

- ... the only way to see your source if you “beat down” the noise

- Noise: $T_{rms} = \frac{T_{sys}}{\sqrt{N}}$



rms fluctuations in the system temperature

fundamental property of the system

number of data points

- Telescope: $N = \Delta\nu\tau$

Radiometer Equation

- So we finally arrive

$$\frac{S}{N} = \frac{T}{T_{rms}} = \frac{T}{\frac{T_{sys}}{\sqrt{\Delta\nu\tau}}} = \frac{T}{T_{sys}} \sqrt{\Delta\nu\tau}$$

- We can re-write this in terms of flux density (rms flux density variations):
SEFD -> System equivalent flux density (Jy) -> fundament. prop. telescope

$$S_{rms} = \frac{SEFD}{\sqrt{\Delta\nu\tau}}$$

longer we integrate & more bandwidth -> higher our S/N and the lower our flux density variations

- We can also extend this to interferometers (N dishes):

$$S_{rms} = \frac{SEFD}{\sqrt{\frac{N(N-1)}{2} \tau 2\Delta\nu}} = \frac{SEFD}{\sqrt{N(N-1)\tau\Delta\nu}}$$

Radiometer Equation



- So we can see that it really is all about S/N
- The more dishes you have the longer you integrate ... and the bigger your bandwidth is
- the better you will do
- The smallest change in antenna temperature T_{\min} that can realistically be detected is normally taken as three times the rms noise (T_{rms})