A brief introduction to radio interferometry I

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  - Perley, Myers (NRAO synthesis imaging summer school).
  - Swinburne University of Technology, Introduction to radio astronomy.
  - M. Garrett, Radio astronomy lecture, 2013
Radio telescopes

- Radio telescopes are designed to detect natural radio emission from objects beyond the Earth.
- The most common simple telescope has a parabolic dish reflector with associated feed(s).
Power and Intensisty

- Consider a distance source of brightness distribution $I_\nu(\hat{s})$
- Power form this emission passes through a sensor
- The power $dp$ from a small solid angle $d\Omega$ is: $dp = I_\nu \nu dA d\Omega$
- The total power received is an integral over frequency, area and angle accounting for variation in responses
Angular resolution: single dish

- The angular resolution of a radio telescope measures its ability to discern fine detail in the structure of a radio source.
- For a single radio telescope the angular resolution can be written as:
  \[
  \theta_{\text{rad}} = 1.22 \times \text{wavelength observed} = 1.22 \frac{\lambda}{D}
  \]
  telescope diameter

Remember: 1 degree = 3600 arcseconds and 360 degrees = \(2\pi\) radians

- Example: consider the (currently) largest fully steerable telescope (Green Bank Telescope) with 100 m diameter. At a wavelength of 6 cm the angular resolution is about 2 arcminutes (or 120 arcseconds)
- This is not good enough! Radio sources can show emission on scales of arcminutes \(\rightarrow\) arcseconds \(\rightarrow\) milliarcseconds...
Why interferometry?

- We want high resolution!
- Improving the resolution of a single telescope requires increasing the collecting area --> bigger telescope.
- To obtain 1 arcsecond resolution at a wavelength of 21cm, we would need a ~ 42 km aperture!
- Since this is not practical, we use arrays of smaller telescopes instead and synthesize the equivalent aperture with pairs of antennas --> Interferometry!
Radio interferometry

- It is a technique where the radio signals from two or more radio telescopes are combined. Usually, a number of radio telescopes are linked together to form an array.
- Each telescope in an array can be treated as part of a much larger dish.

Note: it is impractical to build a single dish this big.
Angular resolution: an interferometer

• For a maximum baseline of length B, the angular resolution is given by:

\[ \theta_{\text{rad}} \approx \frac{\lambda}{B} \]

• Most radio interferometers have angular resolutions between 0.1 and 10 arcseconds

In Very Long Baseline Interferometry (VLBI), telescopes located in different parts of the world forming baselines of 1000s of kilometres and get angular resolutions of milli-arcseconds!
What we want to do

- We want to map radio emission from celestial sources

A 21cm single-dish image of the LMC (Staveley-Smith et al.)

A 21cm ATCA observations of the LMC (Kim et al.)
Some more (biased!) examples of radio images made from interferometry observations:

- The center of our Galaxy (332.38 MHz) - NRAO/AUI/NSF
- Synchrotron emission from radio galaxies (5GHz) - NRAO/AVI
- 21-cm HI absorption in radio galaxy - Morganti et al.

Line emission from nearby spiral galaxies - NGC 5055 (M 63), NGC 628 (M 74), NGC 3031 (M 81), NGC 5194 (M 51) - THINGS - The HI Nearby Galaxy Survey.
Interferometry basics: Two-element interferometer

- The origin of interferometry dates back to 1801 - Thomas Young's two-slit experiment

- Waves arriving in phase interfere constructively and waves arriving out of phase interfere destructively.

- If $B << L$ the path difference is just $B \sin \theta$

- The constructive interference occurs when: $B \sin \theta = m\lambda$ (where $m$ is an integer)

- Similarly destructive interference occurs when: $B \sin \theta = (m + 1/2)\lambda$
• For $y << L$, one can approximate: $\sin \theta = y/L$

• So the position of the constructive interference ($y_+$) and destructive ($y_-$) interference are given by:

$$y_+ = m \lambda L / B \quad y_- = (m + 1/2) \lambda L / B$$

• The spacing between successive constructive interference is given by:

$$\Delta y = \lambda L / B$$

• The angular measure of the spacing between the “fringes” is:

$$\theta \sim \lambda / B$$
What’s actually “measured”

- A monochromatic electromagnetic wave of wavelength $\lambda$ can be described by its amplitude and phase, $Ae^{i\phi}$.
- Assume a constant amplitude $A$ with a phase varying as function of time and position.
- If the same wave travels along two paths of different lengths and meet at point $P$, the electric field is:

$$ E = E_1 + E_2 = Ae^{i\phi} + Ae^{i(\phi - \phi_0)} $$

where $\phi_0$ is the phase delay, and corresponds to the pathlength difference, $\tau_0$:

$$ \phi_0 = \frac{2\pi\tau_0}{\lambda} $$
• The brightness on the screen is a time-averaged electric field intensity, $EE^*$:

$$EE^* = (E_1 + E_2)(E_1 + E_2)^*$$

$$= 2A^2 + 2A^2 \cos \phi_0$$

The interfering term

Note: - the only thing that's left here is the phase delay, $\phi_0$
and the resulting amplitude is a purely real quantity → perceived as “brightness”

• The interfering term changes with phase difference
Two-element radio interferometer

- A radio interferometer measures the coherence of the electric field between the 2 receiving antennas.
- Consider a ‘quasi-monochromatic’ radiation, i.e. narrow band centered on frequency $d\nu/\nu<<1$.

The geometric delay:

$$\tau_g = \vec{B} \cdot \hat{S}/c$$

$$|\tau_g| = B \sin \theta / c$$
Our aim

- We want to find a relation between the characteristics of the product of the voltages from the two antennas of the basic interferometer and the brightness distribution of the source emission.

Remember that radio interferometers do not directly provide an image of a source. The radio image, however, can be reconstructed, albeit imperfectly, from the interference patterns measured on the different baselines.
Basic assumptions

• Assumptions to simplification of the problem:
  – Source is very far, i.e. incoming waves are parallel
  – Source is spatially incoherent
  – Consider ‘quasi-monochromatic’ radiation, i.e. narrow band centered on frequency \( \frac{d\nu}{\nu} \ll 1 \). The amplitude and phase remain unchanged to a time duration of order \( dt \sim 1/d\nu \)
  – Distant radio source in direction \( \mathbf{s} \) radiates and produces a time variable electric field \( E_\nu(t) = E \cos(\omega t + \phi) \)
  – The electric fields then form a small solid angle \( d\Omega \) in the direction \( \mathbf{s} \), within some small bandwidth \( d\nu \), at frequency \( \nu \).
Two-element interferometer: combining signals

Incoming radio waves are converted into electrical signals. The signals are then sent to a correlator. The correlator multiplies and averages the signal.

Output voltage $V_2$ is delayed by $\tau_g$ with respect to $V_1$.

\[ V_1 = V \cos(\omega (t - \tau_g)) \]
\[ V_2 = V \cos(\omega t) \]
\[ R = \frac{V_1 V_2}{2} \cos(\omega \tau_g) \]
Modern radio interferometers use cross-correlation:

\[ V_1 = V \cos[\omega(t - \tau_g)] \]
\[ V_2 = V \cos(\omega t) \]
\[ V_1 V_2 = V^2 \cos(\omega t) \cos[\omega(t - \tau_g)] = \frac{V^2}{2} [\cos(2\omega t - \omega \tau_g) + \cos(\omega \tau_g)] \]

The correlator performs some averaging in time (i.e. for \( 2\omega t \gg 1 \), \( \cos(\omega t - \omega \tau_g) \) averages to zero quickly)

The final output \( R \) becomes:

\[ R = \langle V_1 V_2 \rangle = \frac{V^2}{2} \cos(\omega \tau_g) \]

Since \( V^2 \) is proportional to power: \( R \propto P \cos(\omega \tau_g) \)

Note that \( R \) depends only on the received \( P \) and \( \tau_g \)
Examples of signal combination

- Two signal are shown in red and blue, their product is in black. The output response is their product average.

In phase when 
\[ \omega \tau_g = 2\pi m \]

Quadrature phase when 
\[ \omega \tau_g = (2m+1)\pi/2 \]

Out of phase when 
\[ \omega \tau_g = (m+1/2)\pi \]
“Fringe” analysis

- The output response varies as the direction of the source changes (Earth’s rotation). This variation creates ‘interference fringes’
- Fringe amplitude is $V^2/2$
- The phase of the “fringes” is given by: $\phi = 2\pi v \tau g$
- The change in phase as a function of the source angle $\theta$:

  \[
  \frac{d\phi}{d\theta} = \frac{2\pi v}{c} B \cos \theta = 2\pi \left( \frac{B \cos \theta}{\lambda} \right)
  \]

- The “fringe frequency” is then:

  \[
  \frac{d\phi}{dt} = \frac{2\pi (B \cos \theta)}{\lambda} \frac{d\theta}{dt} = \frac{2\pi \omega (B \cos \theta)}{\lambda}
  \]
The rotation of the Earth moves the source across the sky with the complex output of the interferometer.

- Fringe pattern from a fixed 2-element interferometer (E-W orientation) and pointing at one particular position on the sky:

Note: the small source is unresolved by this fringe pattern. The larger source is resolved.
The frequency of the fringe increases with increasing baseline and decreasing wavelength.

While the longer baselines resolve the small-scale structure in the source, the shorter baselines provide information on the larger scale structure.
Source “visibility”

- The output response $R$ of the correlated signal is called the source “visibility”.
- The source visibility has two parts: the visibility amplitude and visibility phase.
- The visibility amplitude encodes source shape and flux density, while visibility phase encodes source position.
- For a point source the correlated signal varies cosinusoidally between maximum and minimum values.
The response from an extended source

- The sky is composed of spatially incoherent sources (extended sources) with sky brightness distribution, $I_{\nu}(\hat{s})$
- The response of an interferometer towards extended sources is obtained by treating the extended source as sum of individual point sources:
  \[ R_c = \left\langle \int V_1 d\Omega_1 \int V_2 d\Omega_2 \right\rangle \]

This gives us:
\[ R_c = \int I_{\nu}(\hat{s}) \cos(2\pi \vec{B} \cdot \hat{s} / \lambda) d\Omega \]

- This expression relates what we want – the source brightness on the sky $I_{\nu}(\hat{s})$ – to something we can measure – $R_c$ (i.e. the interferometer output response)
- But this response (a cosine correlator) is sensitive only to the even the even part of the sky brightness function
The output from the correlator is actually a complex quantity \((Ae^{i\Phi})\), since we have only been looking at the (even) cosine component of the signal we need to look at the odd component as well.

To measure the odd part we need a sine correlator.

Applying a 90 degree phase shift in one of the signal paths, the correlator’s sine response is:

\[ R_s = \left\langle V_1 V_2 \right\rangle = \frac{V^2}{2} \sin(\omega \tau_g) \]

Combining the "cosine" and "sine" correlators gives a "complex" correlator.
The complex “visibility”

- The output from the correlator is a complex quantity called complex “visibility”

- The complex complex visibility is given as:

  \[
  V_{\nu} = \int I_{\nu} (\hat{s}) \exp(-2\pi i \vec{B} \cdot \hat{s} / \lambda) d\Omega
  \]

- For a small field of view, this expression is a 2-D Fourier transform of the brightness of the sky, and it can be inverted to recover \( I_{\nu}(s) \) from \( V_{\nu} \).

Relationship between the source brightness and the response of an interferometer
The $u$-$v$ plane

- In order to make use of the “visibility” expression (second part of the lecture), it is useful to introduce $u$-$v$ coordinates.
- Each projected baseline (i.e. the baseline as seen from the source) traces out an ellipse with one telescope at the centre of the ellipse.
- And it can be specified using $u$-$v$ coordinates: $u$ gives the east-west component of the baseline. $v$ gives the north-south component of the baseline.

The projected baseline is given by $B \sin \theta = (u^2 + v^2)^{1/2}$.
Interferometer array

- Two-element interferometers measure the positions of unresolved sources to an accuracy comparable to the angular resolution.
- But imaging a more complex radio source requires the use of an array of radio telescopes, i.e. more baselines of different length.
- N number of antennas form N(N-1)/2 baselines, unique to each pair antenna combination.
- The point-source response obtained by averaging the outputs of all antenna pairs is called synthesized beam → aperture synthesis.
Interferometers

ALMA

VLA

WSRT

ATCA

EVN

MeerKAT

GMRT

VLA
Example: u-v coverage and fringe pattern with 2 antennas

Images from Andrea Isella: CASA Radio Analysis Workshop, Caltech
Example: 4 antennas

Images from Andrea Isella: CASA Radio Analysis Workshop, Caltech
Imaging...

“dirty beam”

B(u,v)

“true image”

“dirty image”

Images from David J. Wilner, Harvard CfA
Summary

- The baseline length of an interferometer determines the angular resolution.
- Interferometers measure the interference pattern produced by multiple antennas which can be described in the u-v plane “visibilities”.
- The measured interference patterns can be used to recover information on the source structure and size.
References


- [www.cv.nrao.edu/course/astr534/Interferometers](http://www.cv.nrao.edu/course/astr534/Interferometers)