VLBI Post-Correlation Analysis and Fringe-Fitting

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With (many) Slides from George Moellenbroek and
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HartRAO

Calibration is important!

What Is Delivered by a Synthesis Array?

An enormous list of complex numbers!

E.g., the VLA:

- At each timestamp: 351 [$N*(N-1)/$] baselines (+ 27 auto-correlations)
- For each baseline: 64 Spectral Windows (“IFs”)
- For each spectral window: tens – 1000's of channels
- For each channel: 1, 2, or 4 complex correlations
  - RR or LL or (RR,LL), or (RR,RL,LR,LL)
- With each correlation, a weight value
- Meta-info: Coordinates, field, and frequency info

$N = N_t \text{ x } N_{bl} \text{ x } N_{spw} \text{ x } N_{chan} \text{ x } N_{corr}$ visibilities

- a few x 10^6 x N_{spw} x N_{chan}xN_{corr} vis/hour – 10 to 100s of GB per observations

MeerKAT (64 antennas = 2016 baselines)

VLBI – not quite so bad yet!

Visibility Measurement in Theory

- Formally, we wish to use our interferometer to obtain the visibility function:

$$V(u,v) = \int_{sky} I(l,m) e^{-i2\pi(ul+vm)} dl dm$$

- ….a Fourier transform which we intend to invert to obtain an image of the sky:

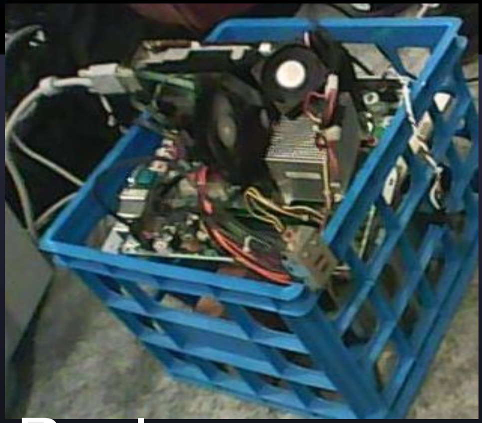
$$I(l,m) = \int_{uv} V(u,v) e^{i2\pi(ul+vm)} du dv$$

  - *V(u,v)* describes the amplitude and phase of 2D sinusoids that add up to an image of the sky
    - Amplitude: "~how concentrated?"
    - Phase: "~where?"

But in Reality....

Weather

Real Clocks



Real electronics

Real antennas

Interference (RFI)

Why Calibration and Editing?

- Synthesis radio telescopes, though well-designed, are not perfect (e.g., surface accuracy, receiver noise, polarization purity, stability, etc.)
- Need to accommodate deliberate engineering (e.g., frequency conversion, digital electronics, filter bandpass, etc.)
- Hardware or control software occasionally fails or behaves unpredictably
- Scheduling/observation errors sometimes occur (e.g., wrong source positions)
- Atmospheric conditions not ideal
- Radio Frequency Interference (RFI)

Why Calibration and Editing?

- Correlator model is good, but not perfect
- Typically, antenna models and locations are now very good, but...
- Source positions are imperfect, and can vary with time and frequency
- Atmosphere and ionosphere are time-variable and unpredictable
- clock information has significant errors at the VLBI level of accuracy

- Determining *instrumental properties* (calibration) is a prerequisite to determining *radio source properties*

Radio Frequency Interference (RFI)

RFI originates from man-made signals generated in the antenna electronics or by external sources (e.g., satellites, cell-phones, radio and TV stations, automobile ignitions, microwave ovens, computers and other electronic devices, etc.)

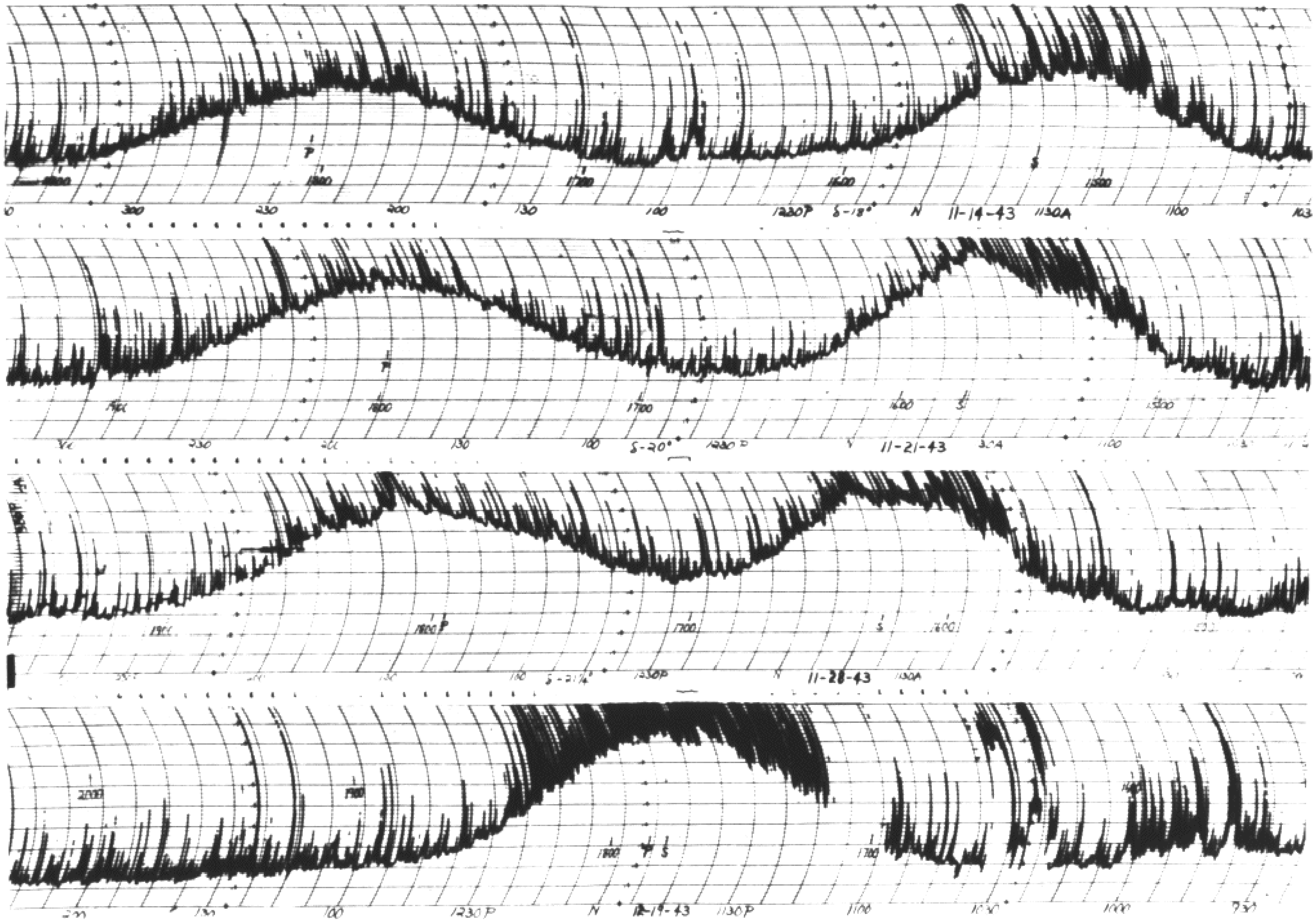
  Adds to total noise power in all observations, thus decreasing the fraction of desired natural signal passed to the correlator, thereby reducing sensitivity and possibly driving electronics into non-linear regimes

  Can correlate between antennas if of common origin and baseline short enough (insufficient decorrelation via geometry compensation), thereby obscuring natural emission in spectral line observations

Least predictable, least controllable threat to a radio astronomy observation

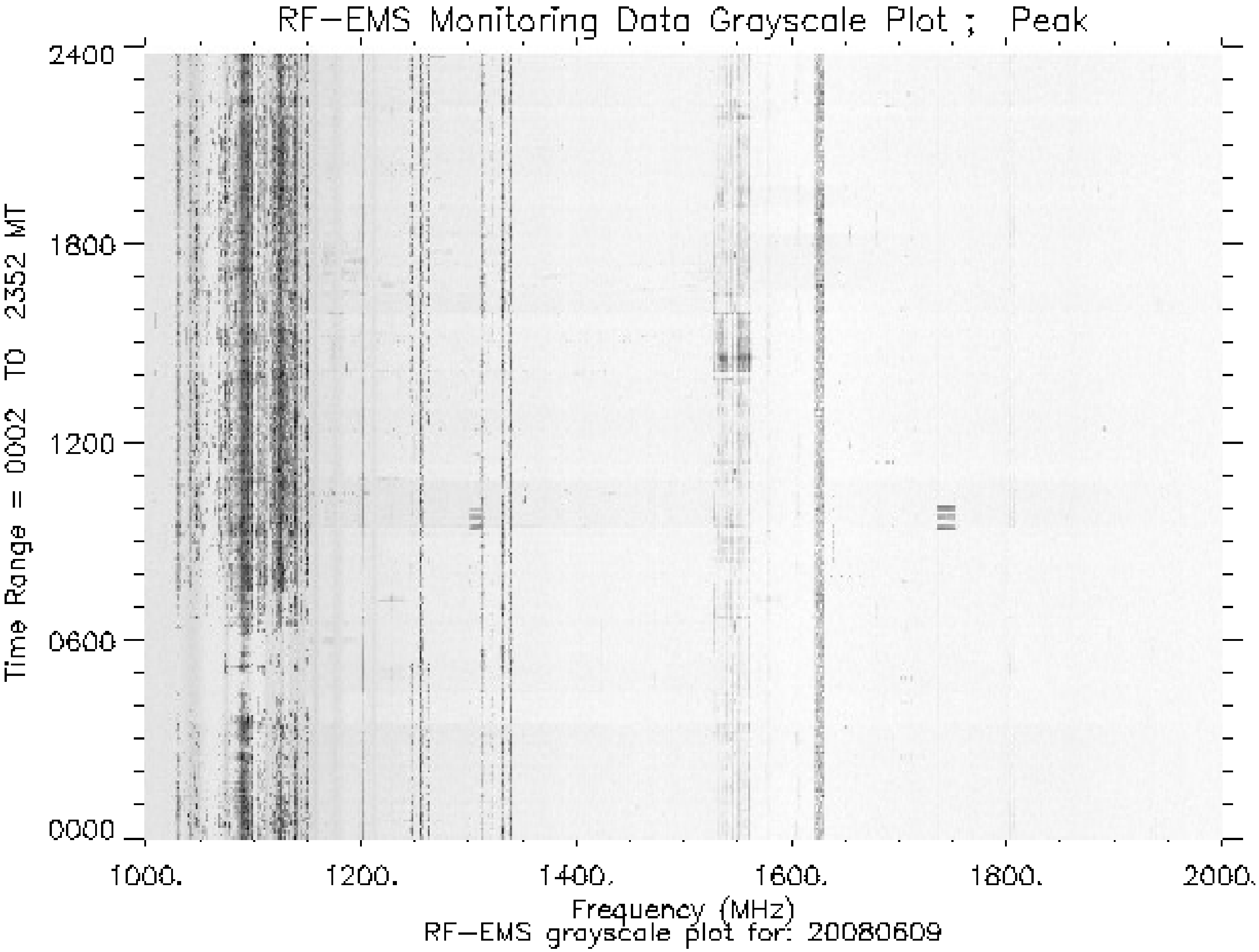
Radio Frequency Interference

Has always been a problem (Reber, 1944, in total power)!



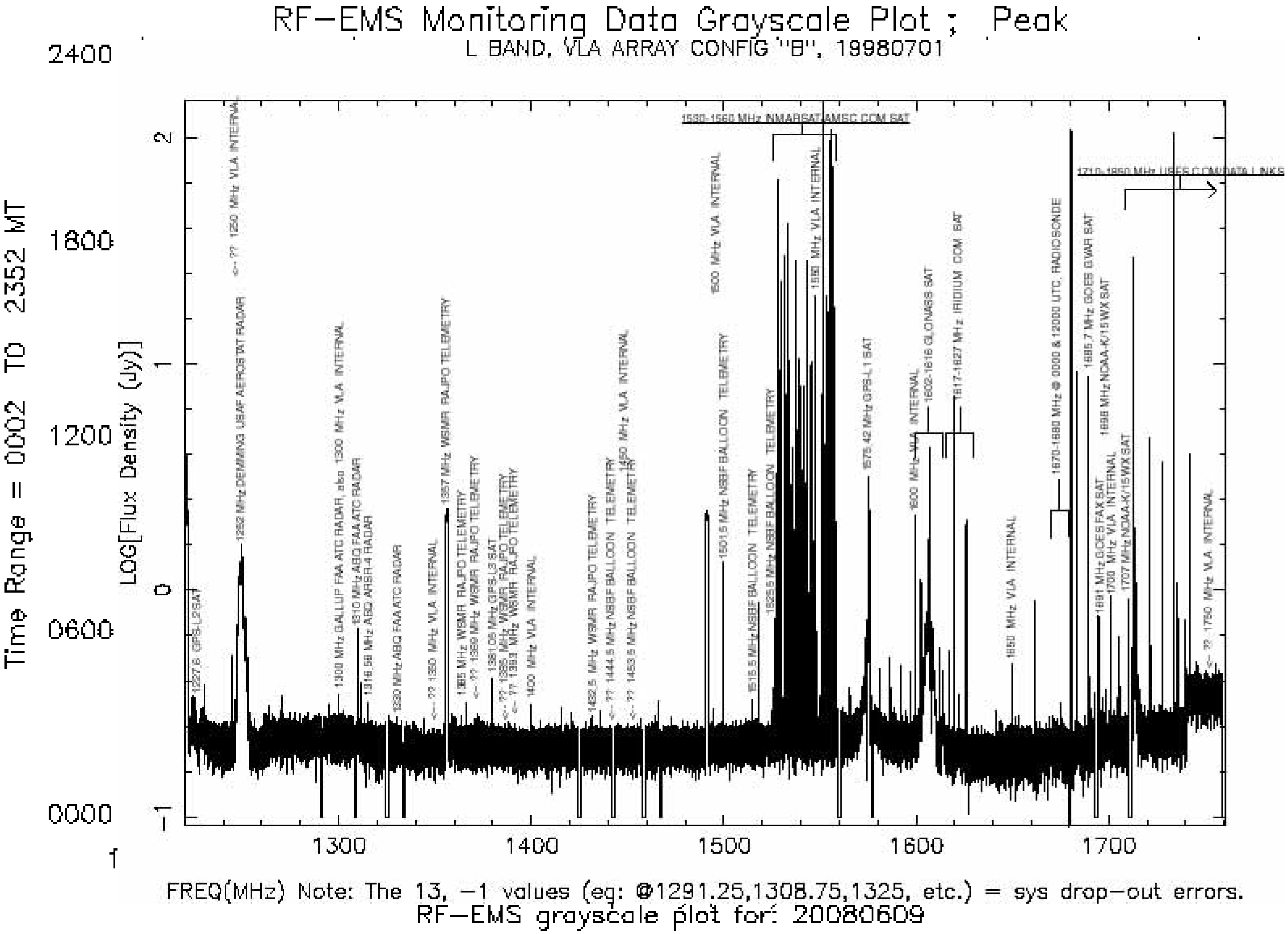
Radio Frequency Interference (cont)

Growth of telecom industry threatening radio astronomy!

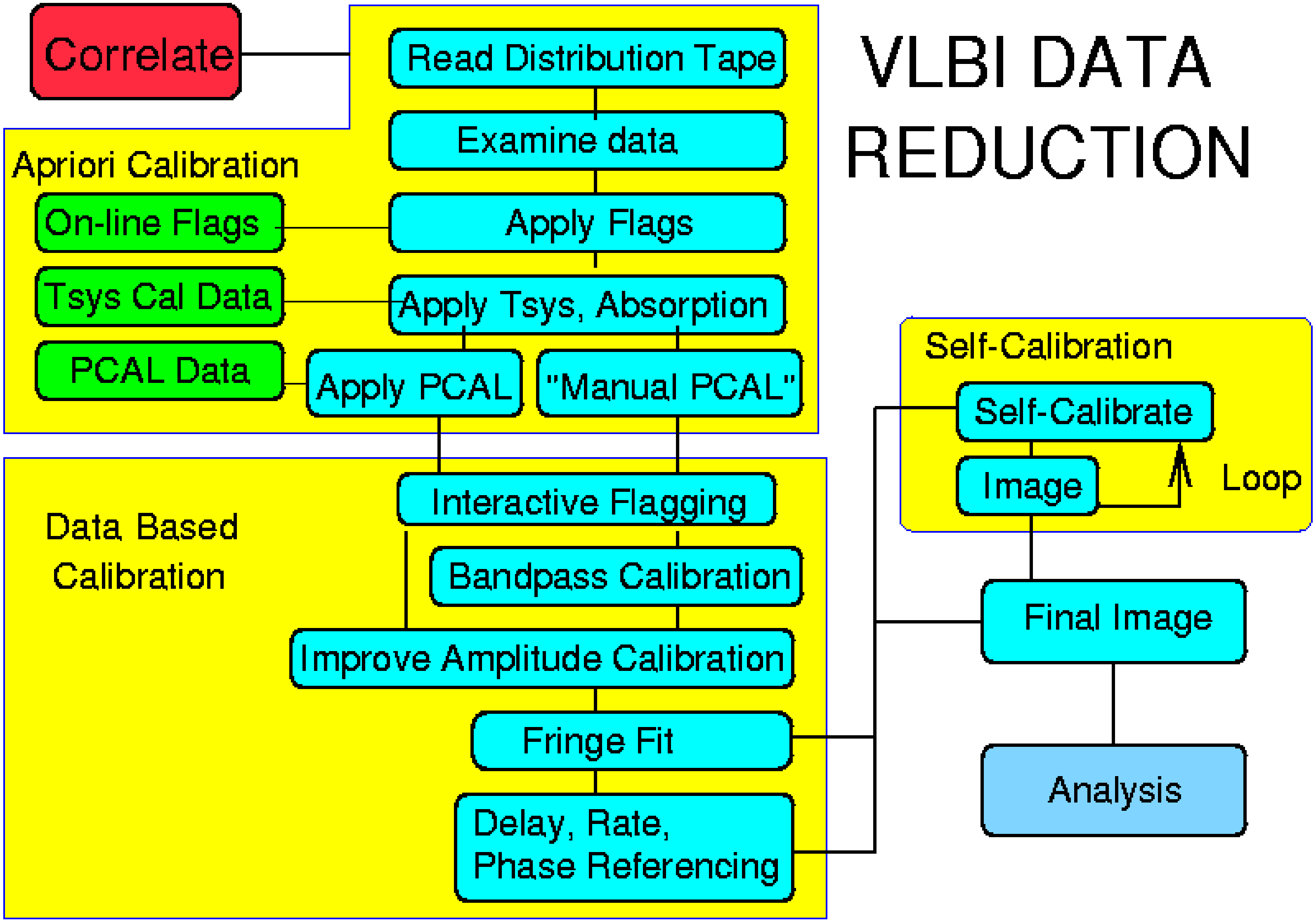


Radio Frequency Interference (cont)

Growth of telecom industry threatening radio astronomy!



VLBI DATA REDUCTION



Correlate

Apriori Calibration

Read Distribution Tape

Examine data

On-line Flags

Apply Flags

Tsys Cal Data

Apply Tsys, Absorption

PCAL Data

Apply PCAL

"Manual PCAL"

Data Based Calibration

Interactive Flagging

Bandpass Calibration

Improve Amplitude Calibration

Fringe Fit

Delay, Rate, Phase Referencing

Self-Calibration

Self-Calibrate

Image

Loop

Final Image

Analysis

Practical Calibration Considerations

A priori “calibrations” (provided by the observatory)

- Antenna positions, earth orientation and rate
- Clocks, frequency reference
- Antenna pointing/focus, voltage pattern, gain curve
- Calibrator coordinates, flux densities, polarization properties
- T_{sys}, nominal sensitivity

Absolute *engineering* calibration (dBm, K, Volts)?

- Very difficult, requires heroic efforts by observatory scientific and engineering staff
- **Amplitude:** T_{sys}, or switched-power monitoring to enable calibration to nominal K, or Jy with antenna efficiency information
- Phase: inject phase-cal, water vapor radiometer (ALMA)
- Traditionally we concentrate instead on ensuring instrumental *stability* on adequate timescales

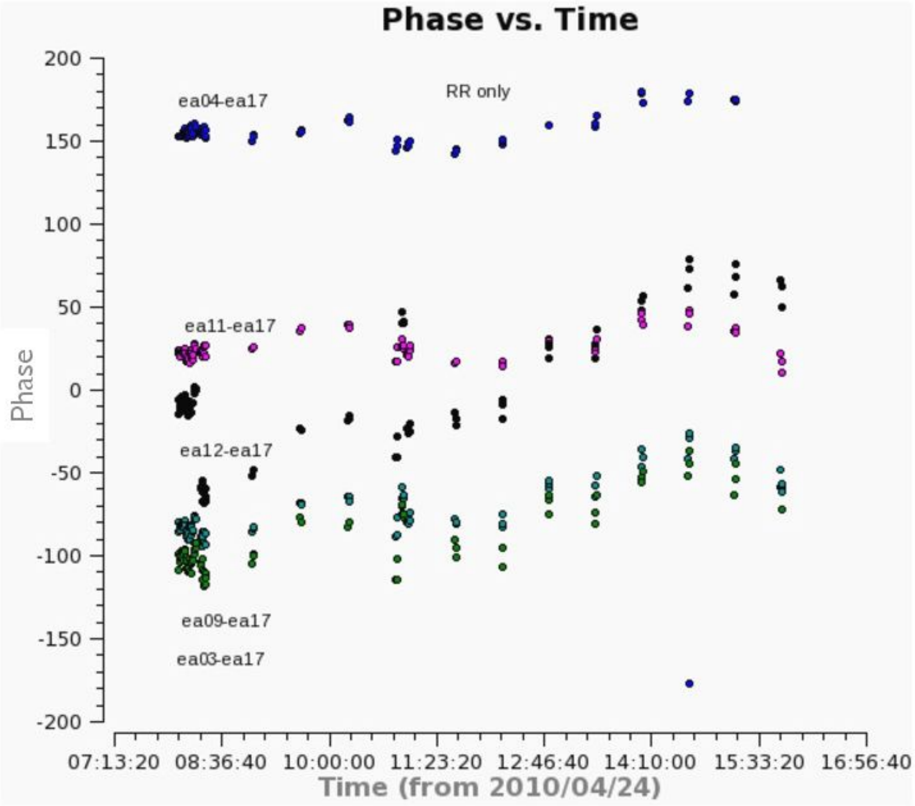
Practical Calibration: Cross Calibration

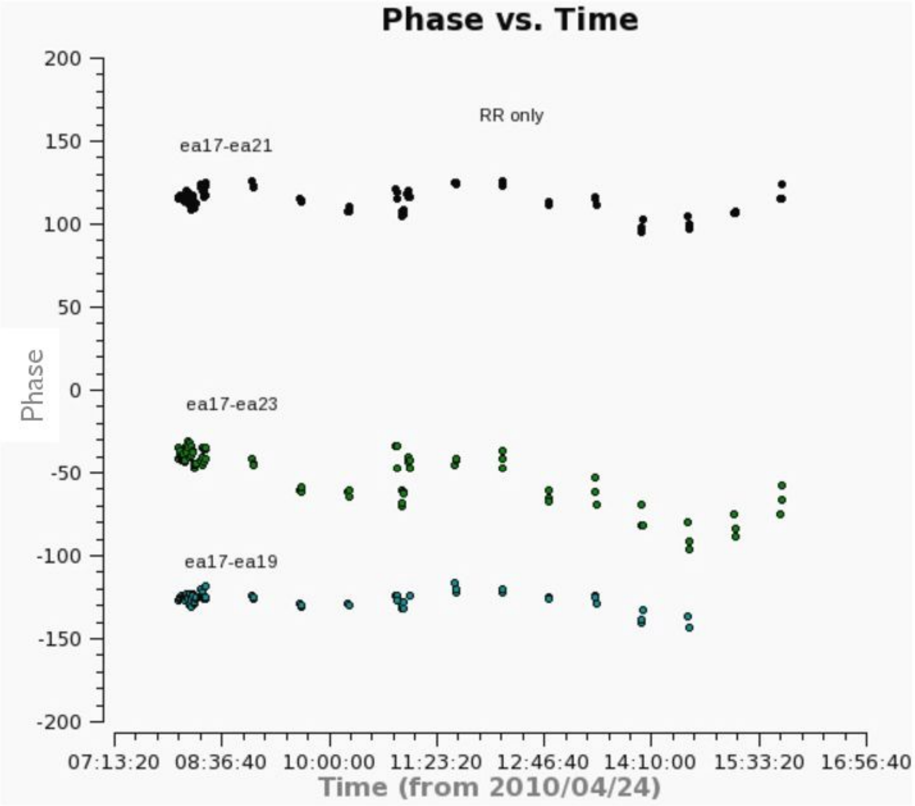
- ***Cross-calibration*** a better choice
  - Observe strong sources – calibrator sources or just calibrators - near the science target whose characteristics (position, flux density) are known!
  - solve for calibration against calibrators and transfer solutions to target observations
  - Choose appropriate calibrators; usually strong point sources because we can easily predict their visibilities: amplitude = constant, phase = 0
    - VLBI: not so easy! most sources somewhat resolved
  - Choose appropriate timescales for calibration (typically minutes; usually longer at low frequencies, shorter at high frequencies)

Antenna-based Cross Calibration

- Measured visibilities are formed from a product of *antenna-based* signals – we can take advantage of this:
- N antennas, there are $N_{\text{baseline}} = N^*(N-1)/2 \sim N^2/2$ baselines.
- Take calibration factor for baseline i,j *to* be $G_{ij,}$ so you need to determine N_{baseline} factors $G_{ij,}$
- If calibration factors into antenna-based factors. so calibration for baseline i,j then $G_{ij,} = G_i \, x \, G_{j,}$ and you need only N factors G_i - much easier if N is large
- Luckily many effects *are* antenna dependent – that is they effect all baselines to any antenna (at some given time) the same way.

Rationale for Antenna-Based Solution





Antenna-based Calibration and Closure

- Success of synthesis telescopes relies on antenna-based calibration
  - Fundamentally, any information that can be factored into antenna-based terms, could *be* antenna-based effects, and not source visibility
  - For $N_{ant} > 3$, source visibility information cannot be *entirely* obliterated by any antenna-based calibration
- Observables independent of antenna-based calibration:
  - Closure phase (3 baselines):

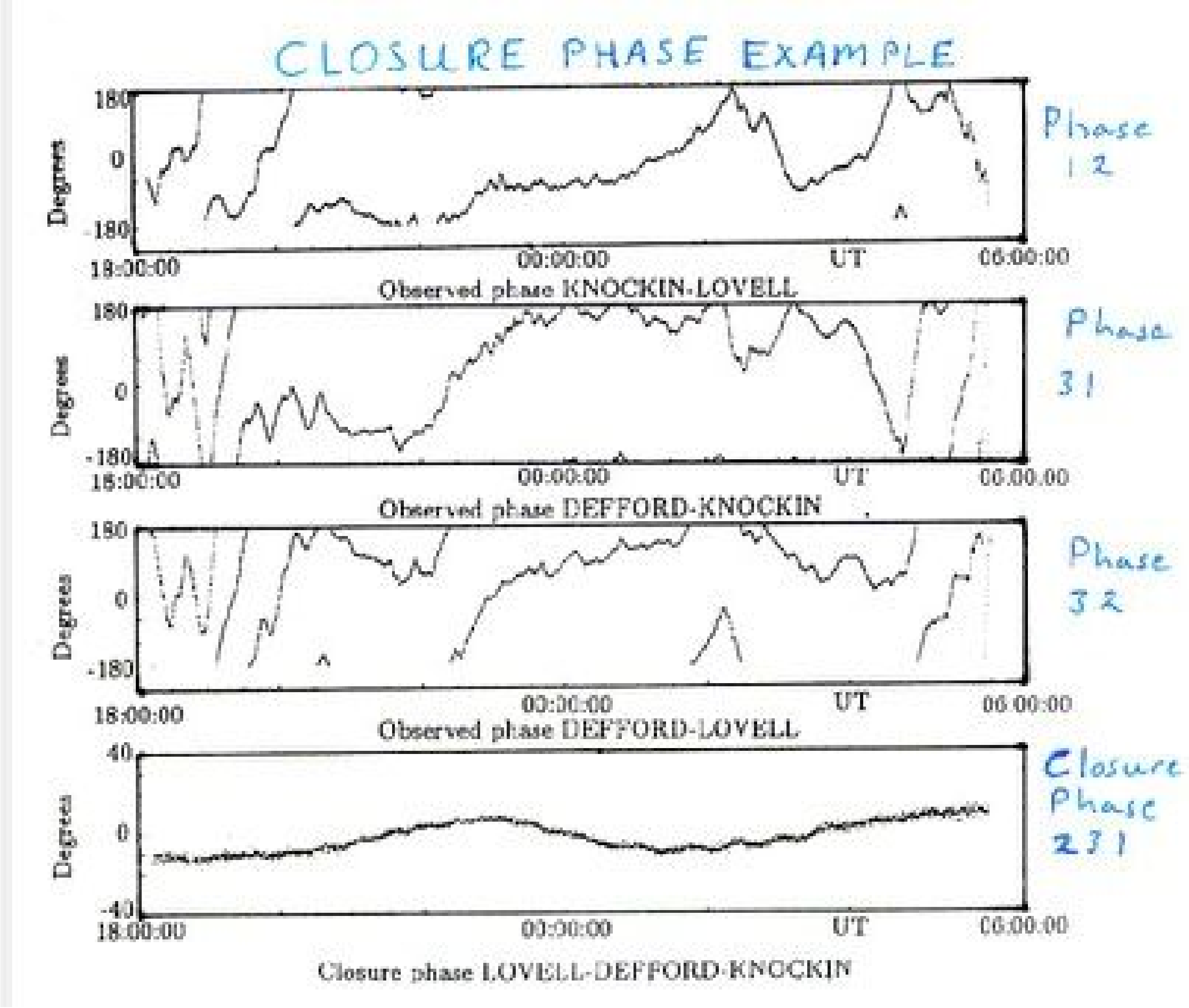
$$\phi_{ij}^{obs} + \phi_{jk}^{obs} + \phi_{ki}^{obs} = \left(\phi_{ij}^{true} + \theta_i - \theta_j\right) + \left(\phi_{jk}^{true} + \theta_j - \theta_k\right) + \left(\phi_{ki}^{true} + \theta_k - \theta_i\right)$$

$$= \phi_{ij}^{true} + \phi_{jk}^{true} + \phi_{ki}^{true}$$

  - Closure amplitude (4 baselines):

$$\left|\frac{V_{ij}^{obs} V_{kl}^{obs}}{V_{ik}^{obs} V_{jl}^{obs}}\right| = \left|\frac{J_i J_j V_{ij}^{true} J_k J_l V_{kl}^{true}}{J_i J_k V_{ik}^{true} J_j J_l V_{jl}^{true}}\right| = \left|\frac{V_{ij}^{true} V_{kl}^{true}}{V_{ik}^{true} V_{jl}^{true}}\right|$$

Closure Phase Example



Observed phase KNOCKIN-LOVELL

Observed phase DEFFORD-KNOCKIN

Observed phase DEFFORD-LOVELL

Closure phase LOVELL-DEFFORD-KNOCKIN

illustration: Tim Cornwell

VLBI Amplitude Calibration

$$S_{cij} = \rho \frac{A}{\eta_s} \sqrt{\frac{T_{si} T_{sj}}{K_i K_j e^{-\tau_i} e^{-\tau_j}}}$$

S_{cij} = Correlated flux density on baseline i - j

ρ = Measured (normalized) correlation coefficient (amplitude 0 to 1)

A = Correlator specific scaling factor

η_s = System efficiency including digitization losses

T_s = System temperature

Includes receiver, spillover, atmosphere, blockage

K = Gain in degrees K per Jansky

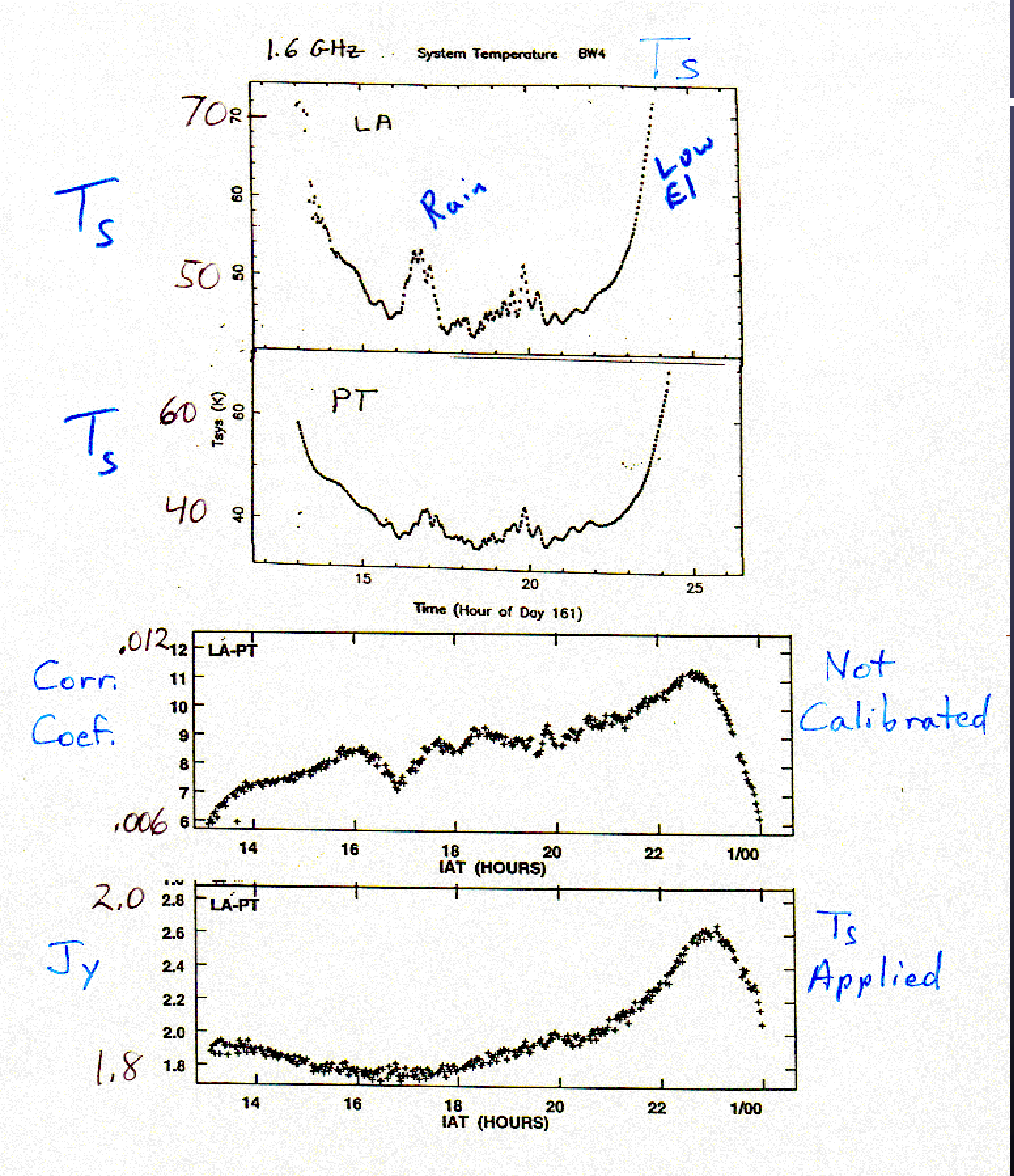
Includes dependence of antenna gain on elevation

$e^{-\tau}$ = Absorption in atmosphere

Note T_s/K = *SEFD* (System Equivalent Flux Density)

Calibration with Tsys

Example shows removal of effect of increased Tsys due to rain and low elevation



Calibration

The measured visibility V' is related to the source visibility V as

$$\langle E_1 E^2 \rangle = V'(u,v)$$

$$= A'(u,v)\, e^{i[\psi(u,v)]} = g_1 g_2\, A(u,v)\, e^{i[\varphi(u,v)+\phi(u,v)]}$$

$$= g_1 g_2\, e^{i[\phi(u,v)]} \times V(u,v)$$

where ψ is the measured phase, φ is the true source phase and ϕ is phase shiftdue to the electronics, atmosphere and ionosphere

Calibration is to determine $g_1 g_2\, e^{i[\phi(u,v)]}$,where the phase noise is typically antenna based. i.e.

$$\phi(12) = [\phi_e(1) - \phi_e(2)] + [\phi_a(1) - \phi_a(2)] + [\phi_i(1) - \phi_i(2)] \ldots$$

Observe calibrations that are point sources of known flux S and known position ($\varphi = 0$), and the measured

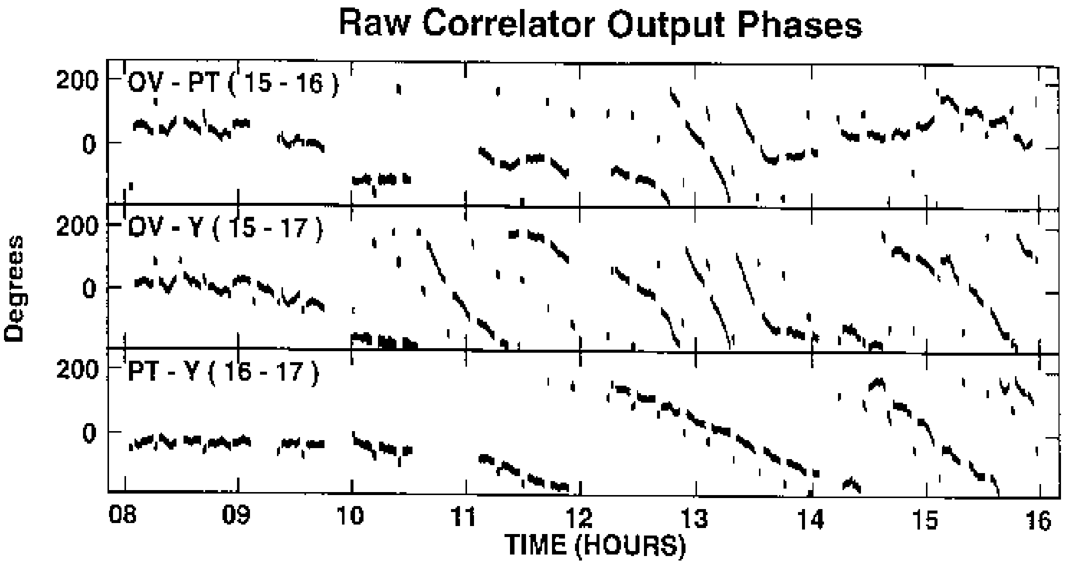
$$V'(u,v)/S = g_1 g_2\, e^{i[\phi(u,v)]} = G_1 G_2^*$$

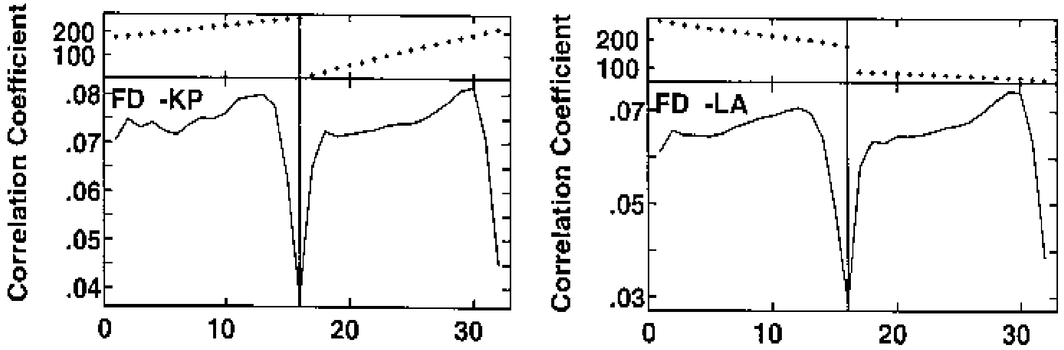
where the complex G represents the amplitude and phase that needs to be removed to yield the true source visibilities.

You measure (phase) calibrators regularly throughout the observations to provide solutions (as a function of time) on N factors G from $N(N-1)/2$ (baseline) measurements. The G(t) are then applied to the observations of the source.

Fringe Fitting

- Raw correlator output has phase slopes in time and frequency
  - Slope in time is “fringe rate”
    - Usually from imperfect troposphere or ionosphere model
  - Slope in frequency is “delay”
    - A phase slope because $\phi=\upsilon\tau$
    - Fluctuations worse at low frequency because of ionosphere
    - Troposphere affects all frequencies equally ("nondispersive")
- Fringe fit is self calibration with first derivatives in time and frequency





Channel (corresponds to frequency)

Why do we need to Fringe Fit?

- Correlator model is good, but not perfect
  - Typically, antenna models and locations are now very good, but...
  - Source positions are imperfect, and can vary with time and frequency
- Atmosphere and ionosphere are time-variable and unpredictable
- Clock information has significant errors at the VLBI level of accuracy

Slide: Olaf Wucknitz

Delay & Rate

- effect of a delay τ
  - ⋆ telescope signal $V_j(t) = A_j \mathrm{e}^{2\pi \mathrm{i} \nu (t-\tau_j)}$
  - ⋆ correlation $\langle V_1 V_2^\star \rangle = A_1 A_2^\star \mathrm{e}^{2\pi \mathrm{i} \nu (\tau_2-\tau_1)}$
  - ⋆ phase $\boxed{\phi = 2\pi\nu(\tau_2 - \tau_1)}$
- frequency dependence
  - ⋆ $\boxed{\dfrac{\partial \phi}{\partial \nu} = 2\pi\tau}$ 'delay' is frequency-derivative of phase
- phase rate and delay rate
  - ⋆ $\boxed{\dfrac{\partial \phi}{\partial t} = 2\pi\nu\dfrac{\partial \tau}{\partial t}}$ equiv. Doppler effect, frequency error

Slide: Olaf Wucknitz

THE DELAY MODEL

Adapted from Sovers, Fanselow, and Jacobs Reviews of Modern Physics, Oct 1998

Item	Approx Max.	Time scale
Zero order geometry.	6000 km	1 day
Nutation	~ 20”	< 18.6 yr
Precession	~ 0.5 arcmin/yr	years
Annual aberration.	20"	1 year
Retarded baseline.	20 m	1 day
Gravitational delay.	4 mas @ 90° from sun	1 year
Tectonic motion.	10 cm/yr	years
Solid Earth Tide	50 cm	12 hr
Pole Tide	2 cm	~1 yr
Ocean Loading	2 cm	12 hr
Atmospheric Loading	2 cm	weeks
Post-glacial Rebound	several mm/yr	years
Polar motion	0.5 arcsec	~ 1.2 years
UT1 (Earth rotation)	Several mas	Various
Ionosphere	~ 2 m at 2 GHz	All
Dry Troposphere	2.3 m at zenith	hours to days
Wet Troposphere	0 – 30 cm at zenith	All
Antenna structure	<10 m. 1cm thermal	—
Parallactic angle	0.5 turn	hours
Station clocks	few microsec	hours
Source structure	5 cm	years

Phase Referencing

- One kind of antenna-based crossed calibration
- Observe a Calibrator source nearby your target
- Calibrator source needs to have accurately known position and ideally be point-like
- Derive calibration (amplitude gains, antenna-phases, rates, delays from calibrator)
- Transfer them to target

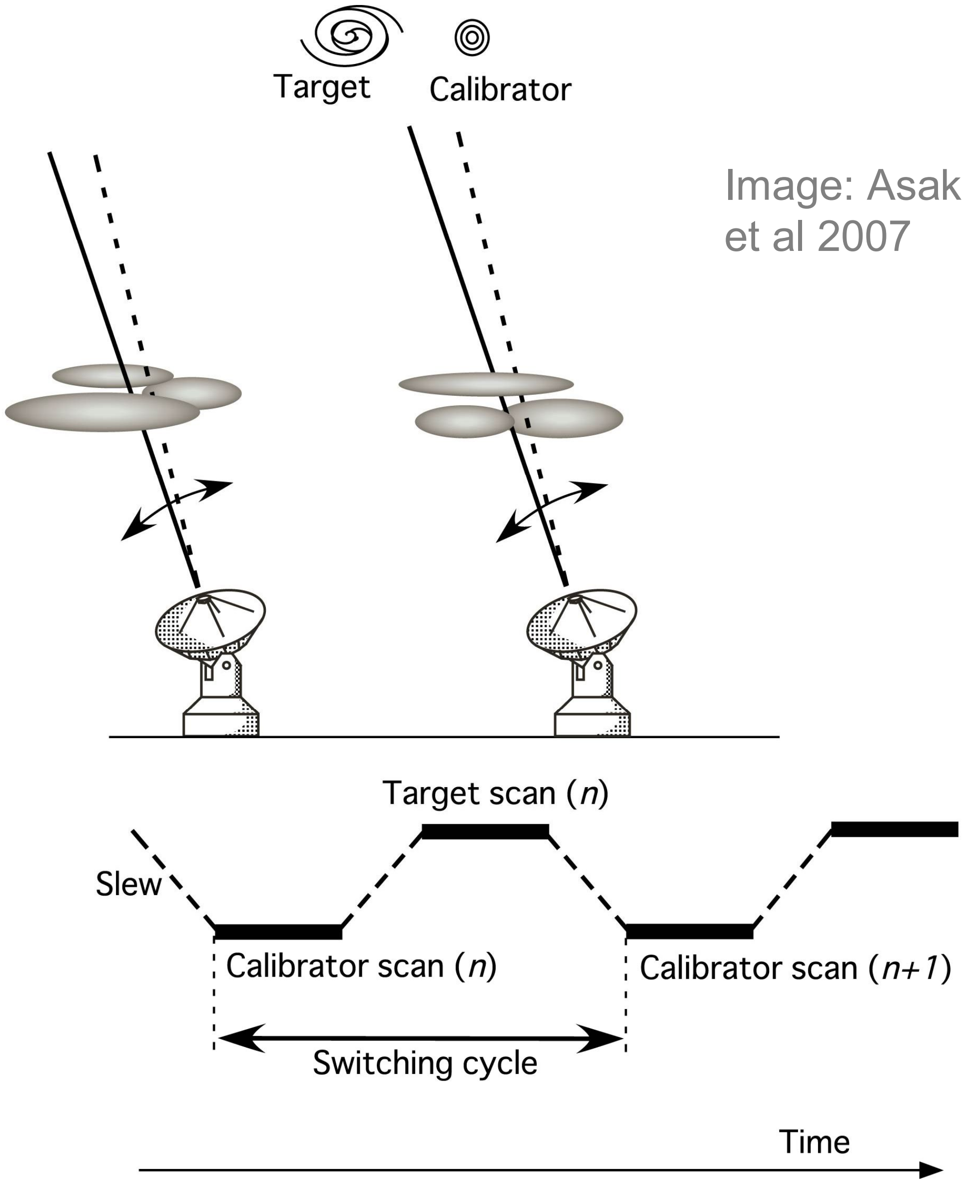
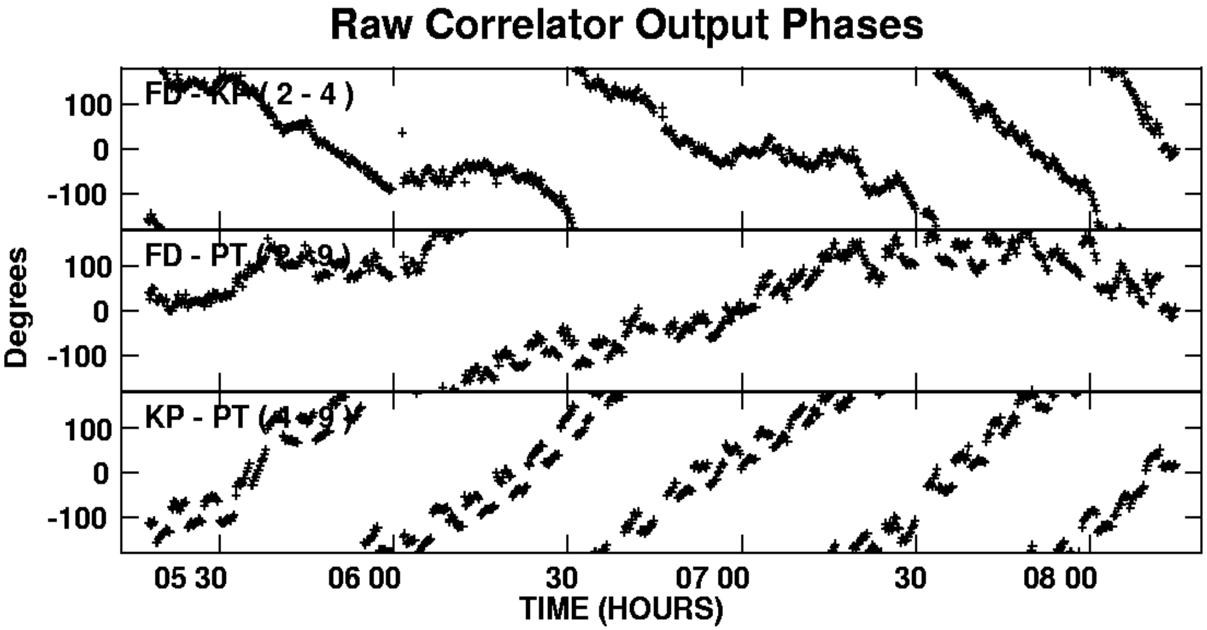


Image: Asaki et al 2007

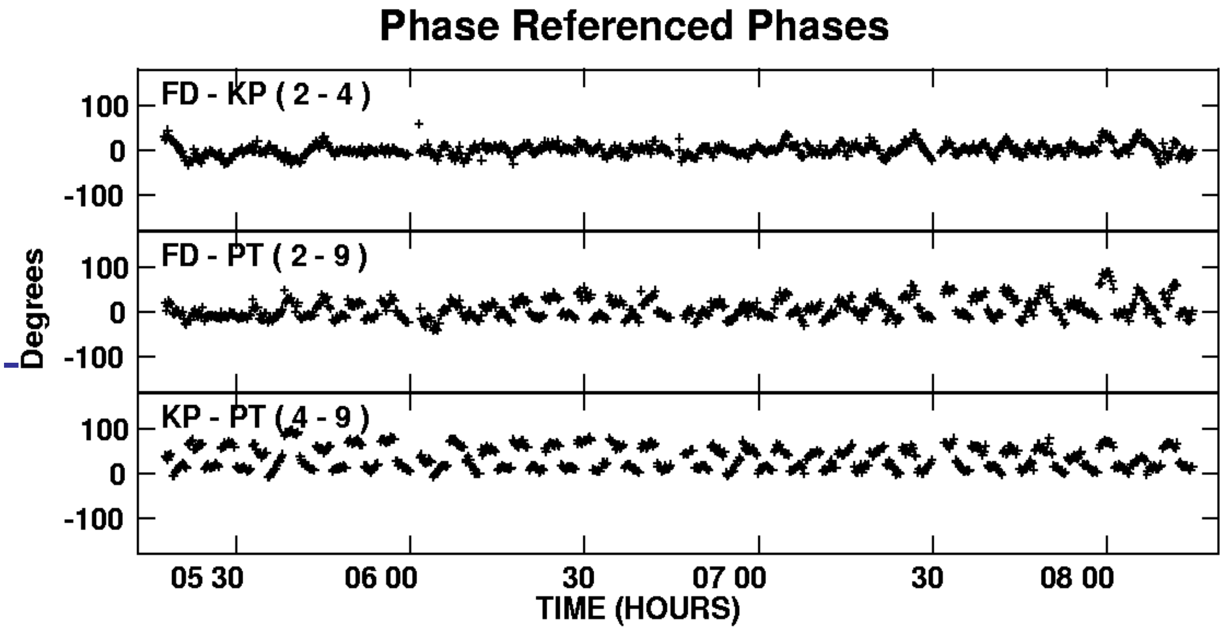
EXAMPLE OF REFERENCED PHASES

- 6 min cycle – 3 min on each source
- Visibility phases of one source were self-calibrated (so after calibration, phases are near zero)
- Phases of the visibilities of the other source phase-shifted by same amount

Raw Correlator Output Phases

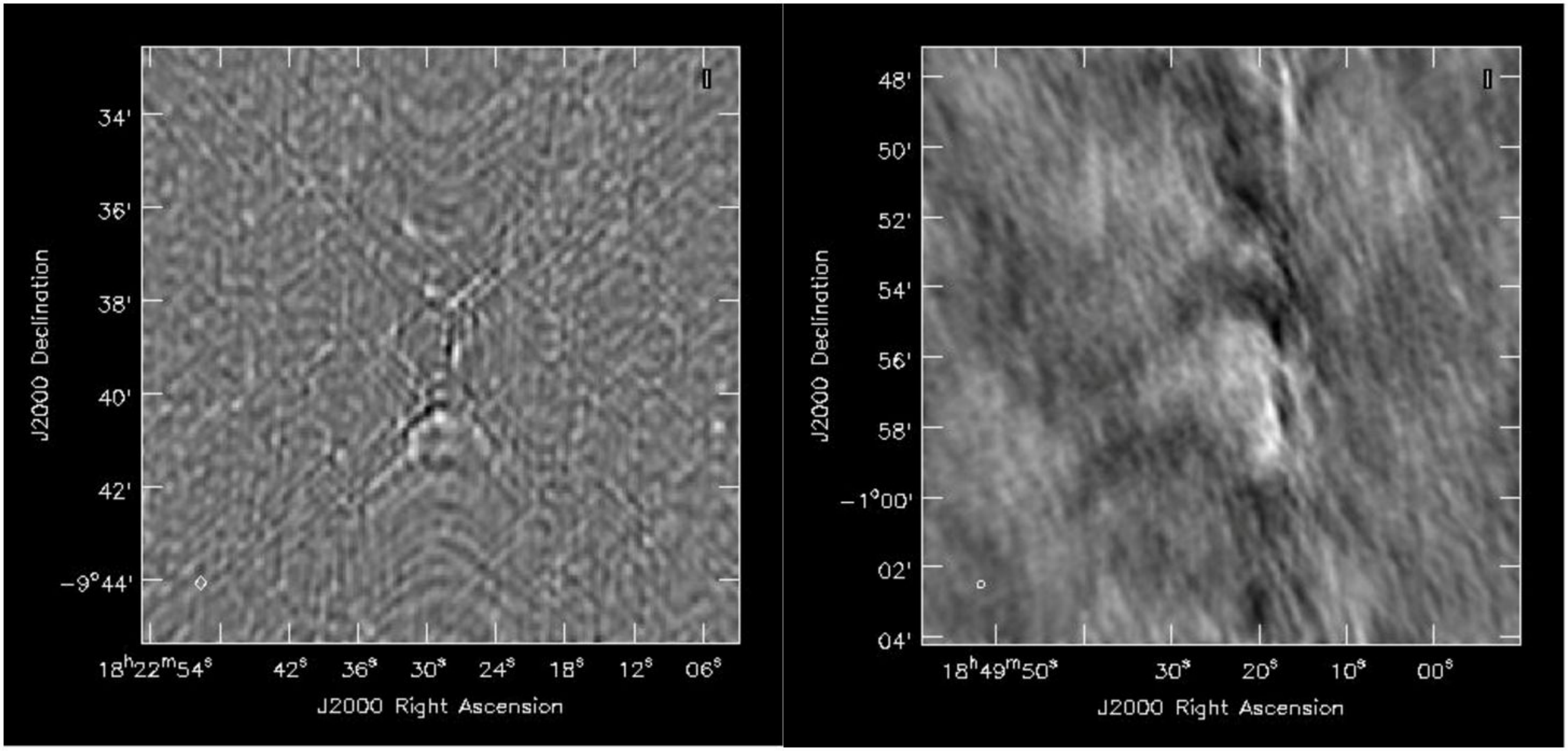


Phase Referenced Phases



Slide: Lo & Cornwel

Effect of Calibration in Images

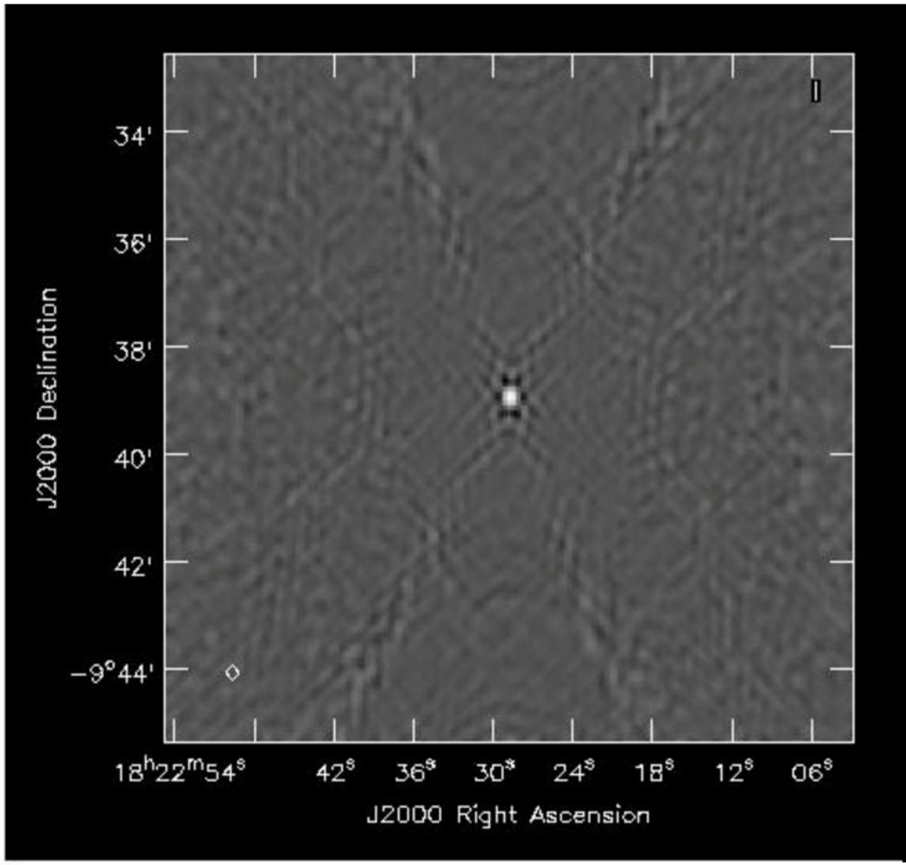


J1822-0938
(calibrator)

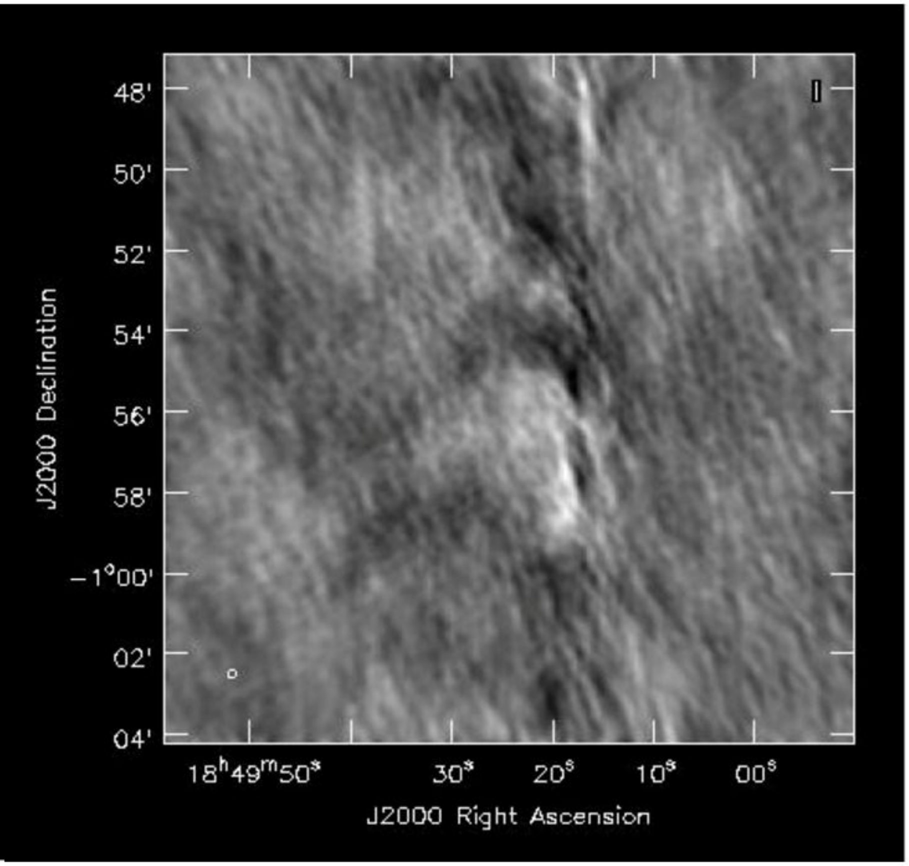
3C391
(science)

Uncalibrated images (VLA) of calibrator J1822-0938 and target 3C391

Effect of Calibration in Images



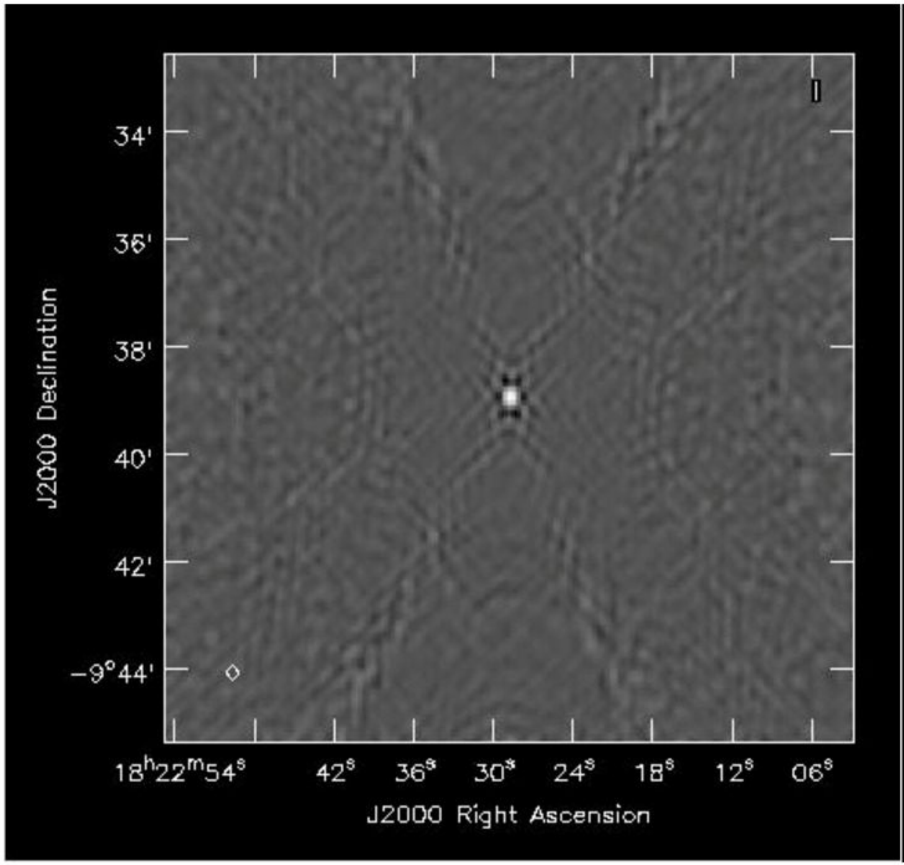
J1822-0938
(calibrator)



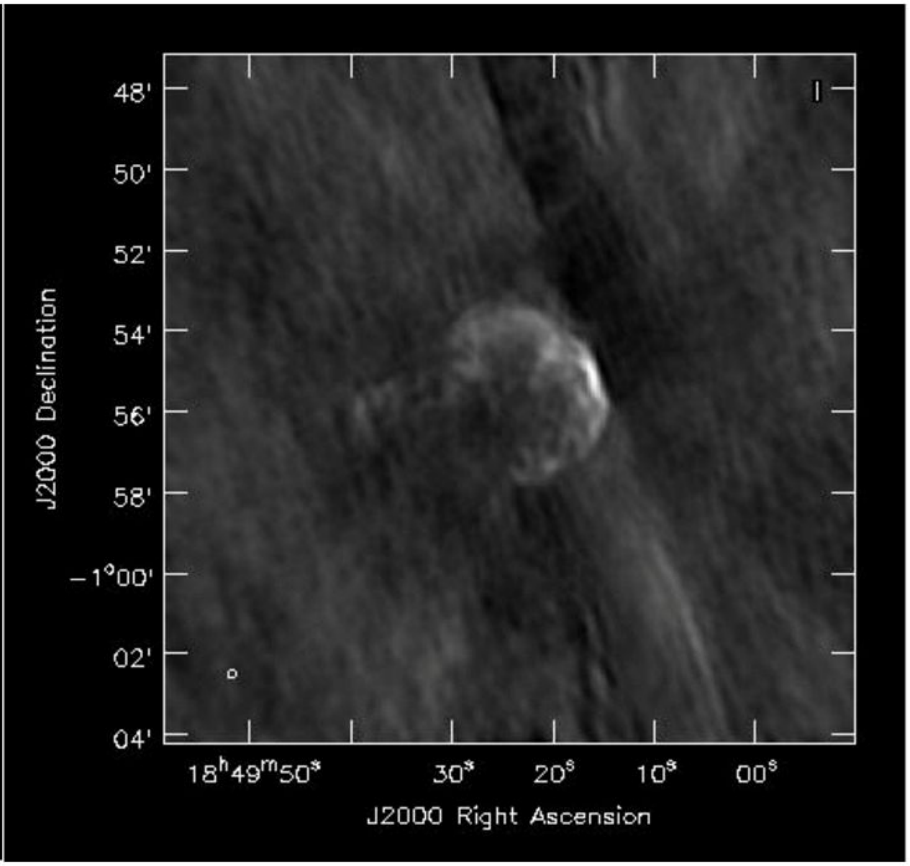
3C391
(science)

Calibrate J1822-0938 (calibrator)

Effect of Calibration in Images



J1822-0938
(calibrator)



3C391
(science)

Transfer calibration solutions to target, 3C391

Summary

Determining calibrations is crucial for getting source properties – you can't have one without the other

Data examination and editing part of the calibration process

Calibration is dominated by antenna-based effects

- permits efficient, accurate and defensible separation of calibration effects from astronomical information (satisfies closure)

Full calibration formalism is complicated, but its modular

Calibration (including editing) is an iterative procedure: improve various properties in turn

Point (unresolved) sources are the best calibrators

Observe calibrators according to the calibration component requirements