TWO-ELEMENT INTERFEROMETER

1 Introduction:

You are going to see a demonstration with a two-element interferometer. It is very simple, it just uses a “mixer” to add the signals from two dishes, and it does not track, so as the sky rotates objects will move through its field of view. The frequency is 12 GHz, and the bandwidth is narrow enough so that you can treat the radiation as mono-chromatic.

We will point it near the sun, since the sun is the brightest radio source in the sky, and it is relatively easy to aim at. As the sun moves across the sky, it will move through the response patterns of the individual antennas and interferometer.

There are two effects here. Firstly, if we look at the output of just one dish: the dish has a diameter 1.2 meters, and so it is sensitive to radiation only from a certain angle on the sky. The response is approximately Gaussian. The detector measures the square of the electric field (so the detector output is always positive) averaged over 0.05 seconds. Since we’re working at 12 GHz, one period is 8.33×10^{-11} seconds, so it is averaging over many, many cycles of the underlying waveform.

Secondly, if we connect both dishes and look at the interferometer output, we’re adding the complex signals from the two antennas and our detector measures the sum of the two signals.

In the mixer, the signals add as vectors, so the two signals could add destructively or constructively — which depends on the time delay between when the signal from the first antenna hits the mixer to the time that the signal from the second one does. This delay will cause second signal to be phase-shifted with respect to the first. If the signals going into the mixer are equal in amplitude and in phase, then the output from the mixer has twice the amplitude of each individual signal. If the signals are exactly out of phase (and equal) then the output of the mixer would be exactly 0.

The first part of that phase shift is just given by the geometry — this is called the geometrical delay. If the line between the two antennas (the baseline) is exactly perpendicular to the direction to the sun then there is no delay difference, and thus no phase shift, and the two signals would add constructively. However, as the sun moves across the sky, the direction to the sun rotates with respect to the baseline, and so there will be some delay. Note that we need only worry about the *difference* in delay between the two antennas, of course the radio waves left the surface of the sun ∼8 minutes ago, but all that matters for us is the much smaller differences between the paths to the two antennas.

In fact, there is a further phase-shift between two antennas because the cables are not of exactly equal length. However, this phase shift remains constant with time.

So as the sun moves, we expect an alternating pattern of maxima and nulls, as the two signals go in and out of phase.

2 Questions:

1. If we only look at the signal from only one dish, and we have the dish pointed exactly at the sun at (local solar) noon, what do you expect the output to look like for the period say from 10 mins before noon to 10 mins after?

2. Now if we add the second dish to make the interferometer, and point it exactly at the sun at noon, with the baseline oriented exactly E-W. How much time do you expect between the two nulls? (Hint: think in terms difference in the geometric delay between the two antennas. At noon, this difference is zero. As the sun moves to the W, the signal will arrive at the W antenna a little earlier than at the E one. The delay depends on the "hour angle" `https://en.wikipedia.org/wiki/Hour_angle`).

3. Now sketch the response we expect out of the interferometer for the period of 20 minutes around noon.

4. Bonus question: What happens when we observe at some time other than noon?

3 Data:

Dish Diameter: 1.2 meters

Frequency: $\nu = 12$ GHz

Speed of light: $c = 3.00 \times 10^8$ meters sec^{-1}

Wavelength $(= c/\nu) = 0.025$ meters

Distance between dish centres: 5.0 meters