VLBI Fundamentals II

Michael Bietenholz

With (many) Slides from George Moellenbroek and
Craig Walker NRAO

HartRAO

What Sources can be observed with VLBI?

Any sufficiently compact radio source can be studied with VLBI

Active Galactic Nuclei (AGN)

Masers

Supernova and (distant) supernova remnants

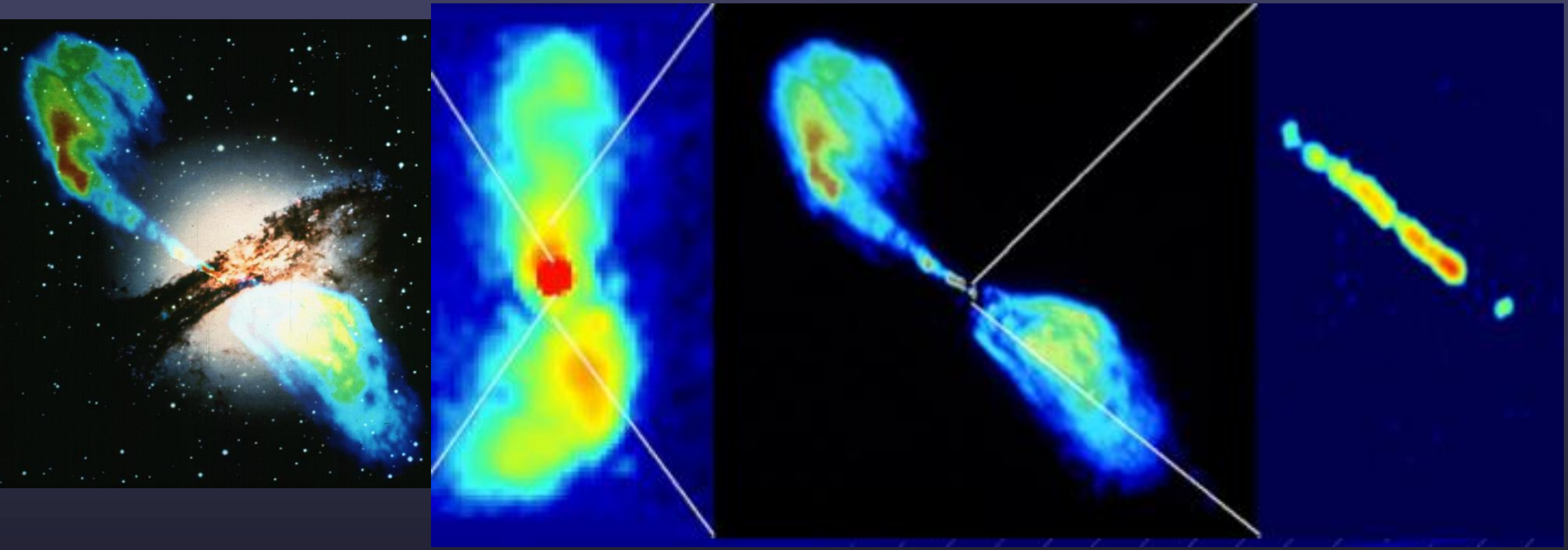
Stars (some)

Pulsars

……

Almost always non-thermal emission – VLBI only sensitive to high brightness temperatures.

Resolution: Centaurus A



Galaxy in the optical with radio (VLA overlay)

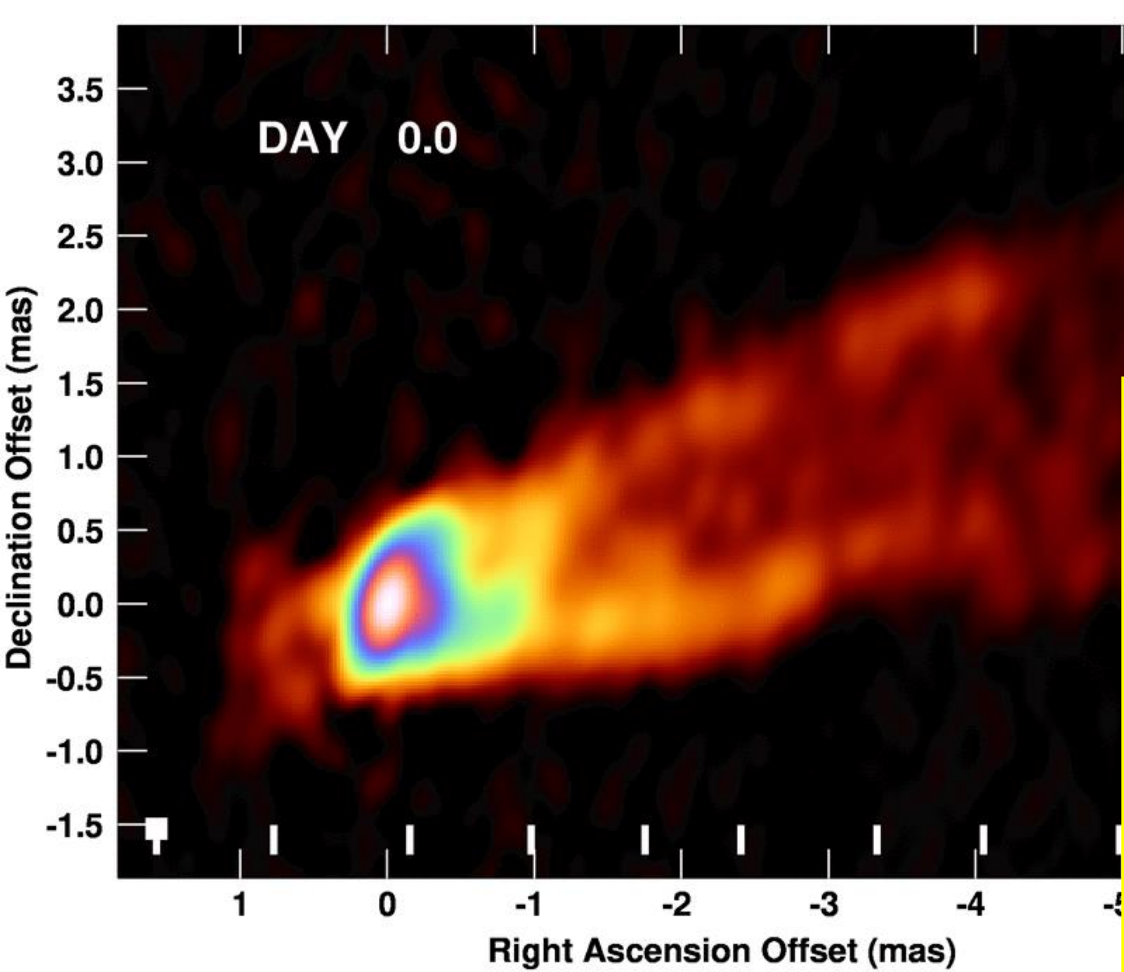
The full radio emission covers nearly 10 degrees on the sky. HartRAO 26m at 13cm, resolution of 20 arcmin

VLA radio continuum observations of the inner lobes a field of view 11 arcmin, resolution ~20 arcsec

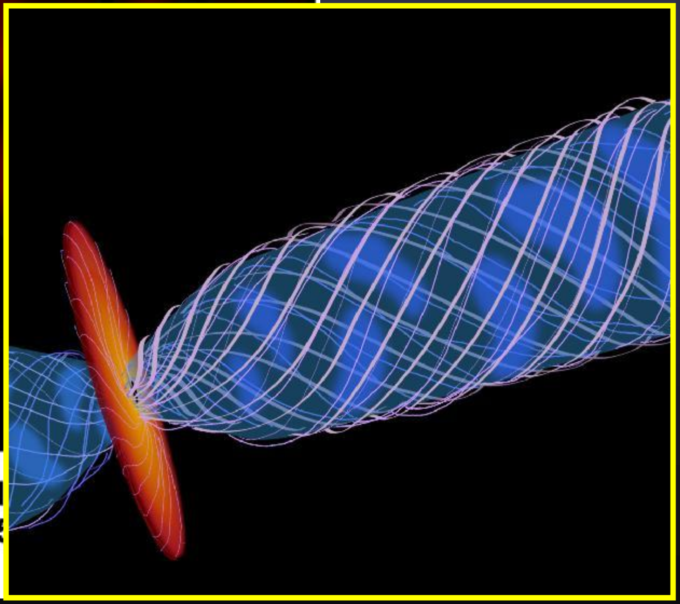
VLBI (LBA + HartRAO) image show fine details of jet near the black hole (centre). Field of view is jet ~0.08 arcsec, resolution is ~0.003 arcsec (milliarcsec)

VLBI Science

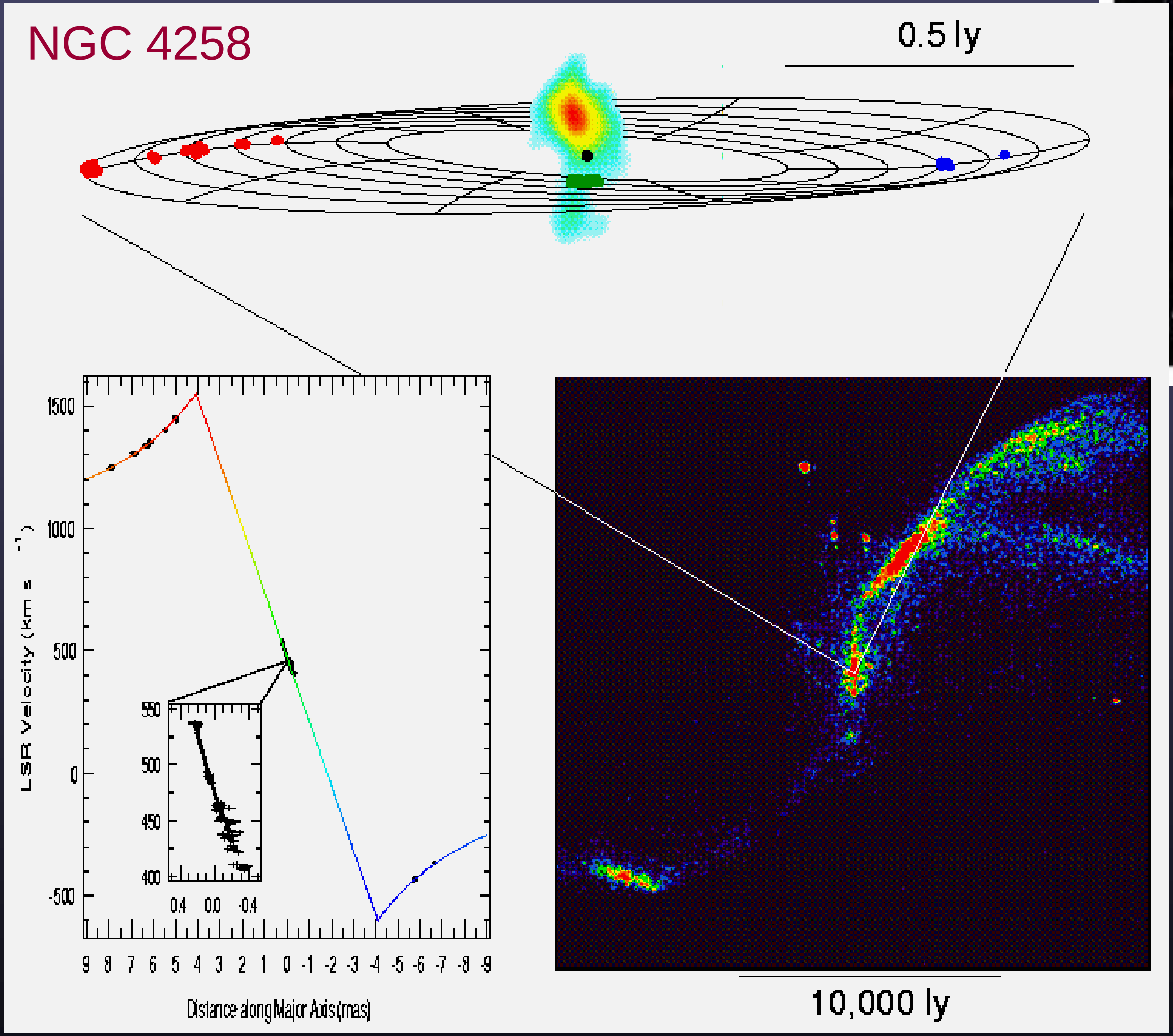
The inner 2 pc of M87 AGN jet (C. Walker et al.) M87 is the dominant galaxy in the Virgo cluster, at ~17 Mpc, and contains a very massive black hole

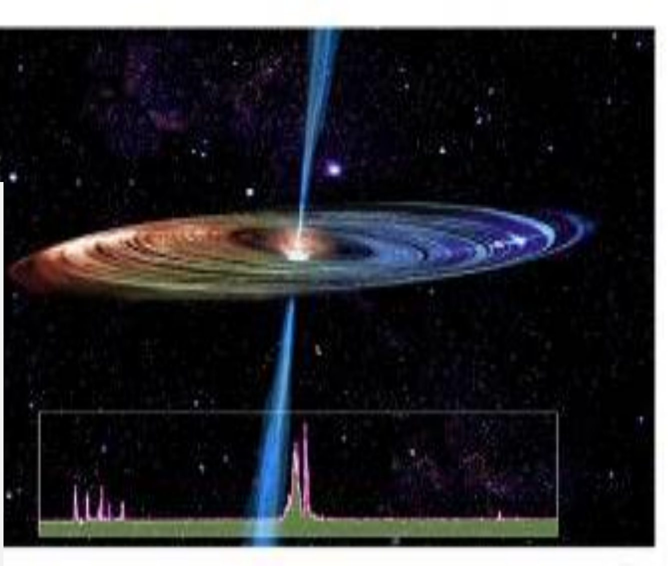

Walker et al. 2016

Black Hole / Jet Model



VLBI Science

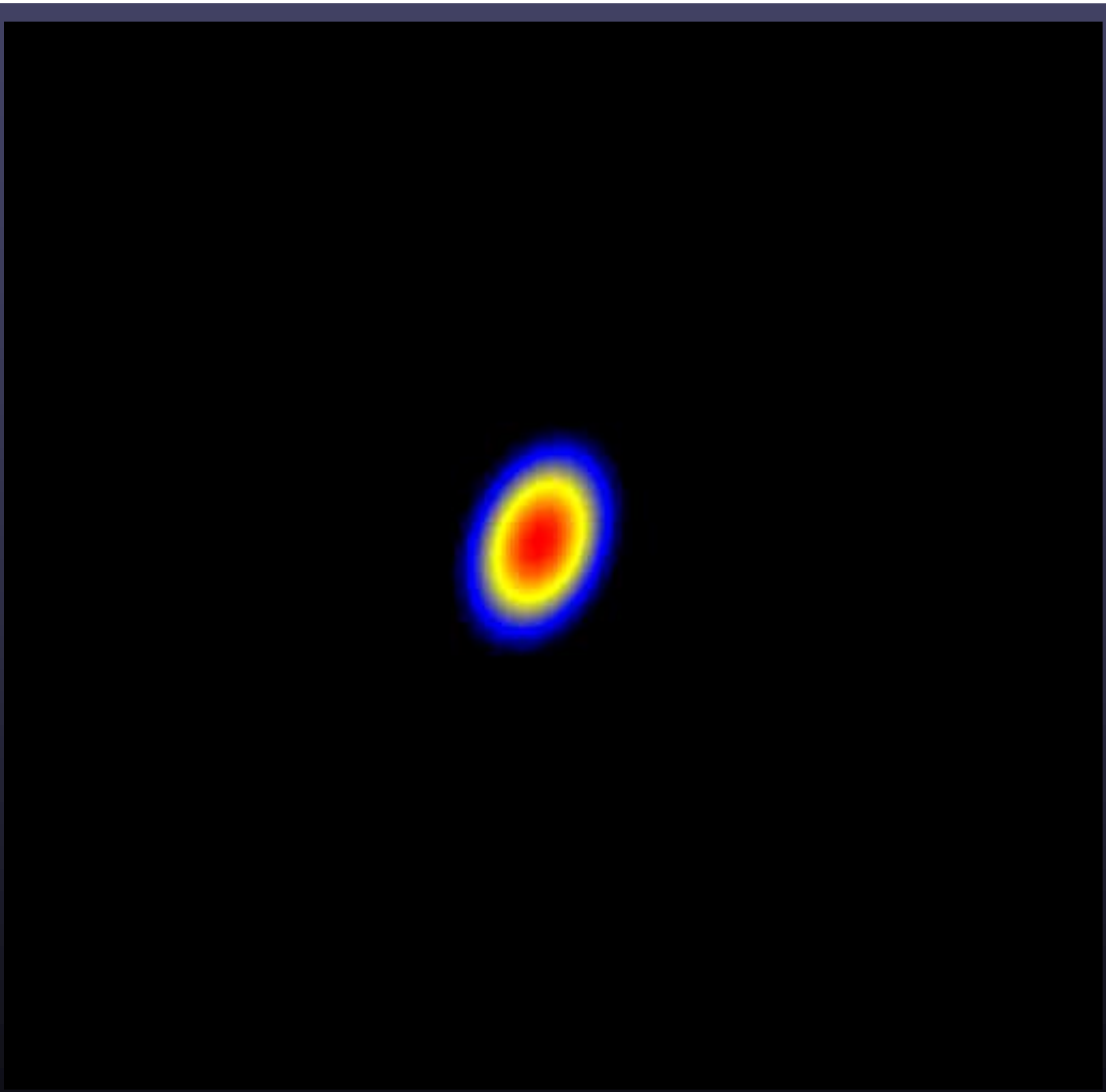




Direct distance :
7.5±0.3 Mpc and
Mass of black hole
$39 \times 10^{6}\ M_{\odot}$

Herrnstein, Greenhill et al

VLBI Science



Bietenholz & Bartel 2017

- VLBI Images: 1987 to 2014 (and continuing…)
- Global VLBI images at 8.4 and 5 GHz

1 milli-arcsecond

Calibration is important!

fotolia

What Is Delivered by a Synthesis Array?

- An enormous list of complex numbers!
- E.g., the Very Long Baseline Array – 10 antennas:
  - At each timestamp: 36 [$N*(N-1)/2$] baselines (+ 10 auto-correlations)
  - For each baseline: 8 Spectral Windows (“IFs”)
  - For each spectral window: tens – 100's of channels
  - For each channel: 1, 2, or 4 complex correlations
    - RR or LL or (RR,LL), or (RR,RL,LR,LL)
  - With each correlation, a weight value
  - Meta-info: Coordinates, field, and frequency info
- $N = N_t \times N_{bl} \times N_{spw} \times N_{chan} \times N_{corr}$ visibilities
  - a few x 10^8 vis/hour – 10 to 100s of GB per observations
- Connected-element interferometers mostly worse:
- VLA: 27 antennas → 351 baselines
- MeerKAT: 64 antennas → 2016 baselines)

Visibility Measurement in Theory

- Formally, we wish to use our interferometer to obtain the visibility function:

$$V(u,v) = \int_{sky} I(l,m) e^{-i2\pi(ul+vm)} dl dm$$

- ….a Fourier transform which we intend to invert to obtain an image of the sky:

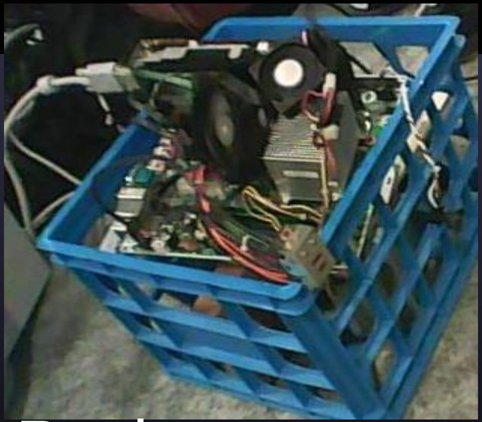
$$I(l,m) = \int_{uv} V(u,v) e^{i2\pi(ul+vm)} du dv$$

  - *V(u,v)* describes the amplitude and phase of 2D sinusoids that add up to an image of the sky
    - Amplitude: "~how concentrated?"
    - Phase: "~where?"

But in Reality....

Weather

Real Clocks



Real electronics

Real antennas

Interference (RFI)

Why Calibration and Editing?

- Synthesis radio telescopes, though well-designed, are not perfect (e.g., surface accuracy, receiver noise, polarization purity, stability, etc.)
- Need to accommodate deliberate engineering (e.g., frequency conversion, digital electronics, filter bandpass, etc.)
- Hardware or control software occasionally fails or behaves unpredictably
- Scheduling/observation errors sometimes occur (e.g., wrong source positions)
- Atmospheric conditions not ideal
- Radio Frequency Interference (RFI)

Why Calibration and Editing?

- Correlator model is good, but not perfect
- Typically, antenna models and locations are now very good, but...
- Source positions are imperfect, and can vary with time, and peak brightness points may vary with frequency
- Atmosphere and ionosphere are time-variable and unpredictable
- clock information has significant errors at the VLBI level of accuracy

- Determining *instrumental properties* (calibration) is a prerequisite to determining *radio source properties*

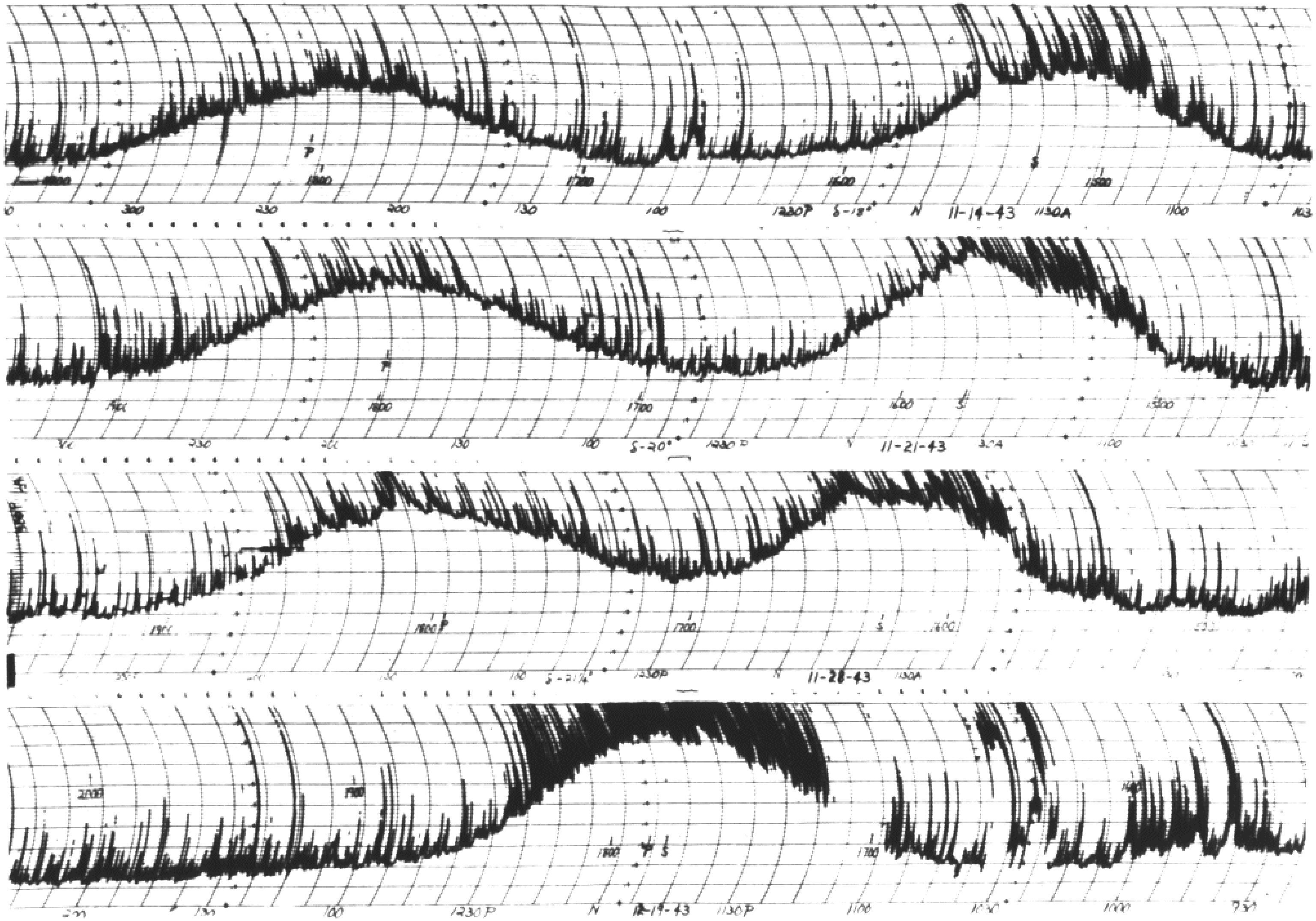
Radio Frequency Interference (RFI)

- RFI originates from man-made signals generated in the antenna electronics or by external sources (e.g., satellites, cell-phones, radio and TV stations, automobile ignitions, microwave ovens, computers and other electronic devices, etc.)
  - Adds to total noise power in all observations, thus decreasing the fraction of desired natural signal passed to the correlator, thereby reducing sensitivity and possibly driving electronics into non-linear regimes
  - Can correlate between antennas if of common origin and baseline short enough (insufficient decorrelation via geometry compensation), thereby obscuring natural emission in spectral line observations

- Least predictable, least controllable threat to a radio astronomy observation

Radio Frequency Interference

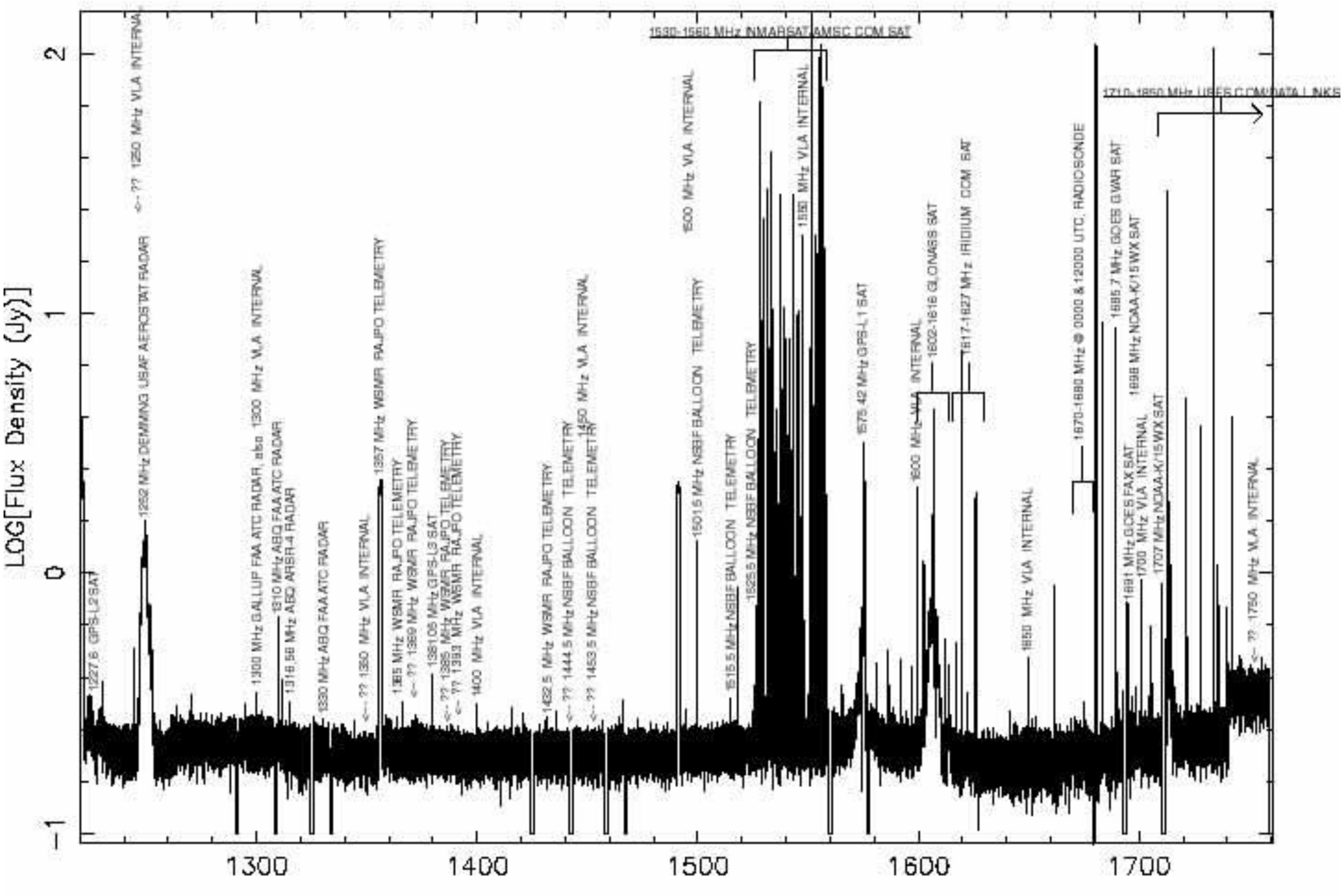
Has always been a problem (Reber 1944, in total power)!



Radio Frequency Interference (cont)

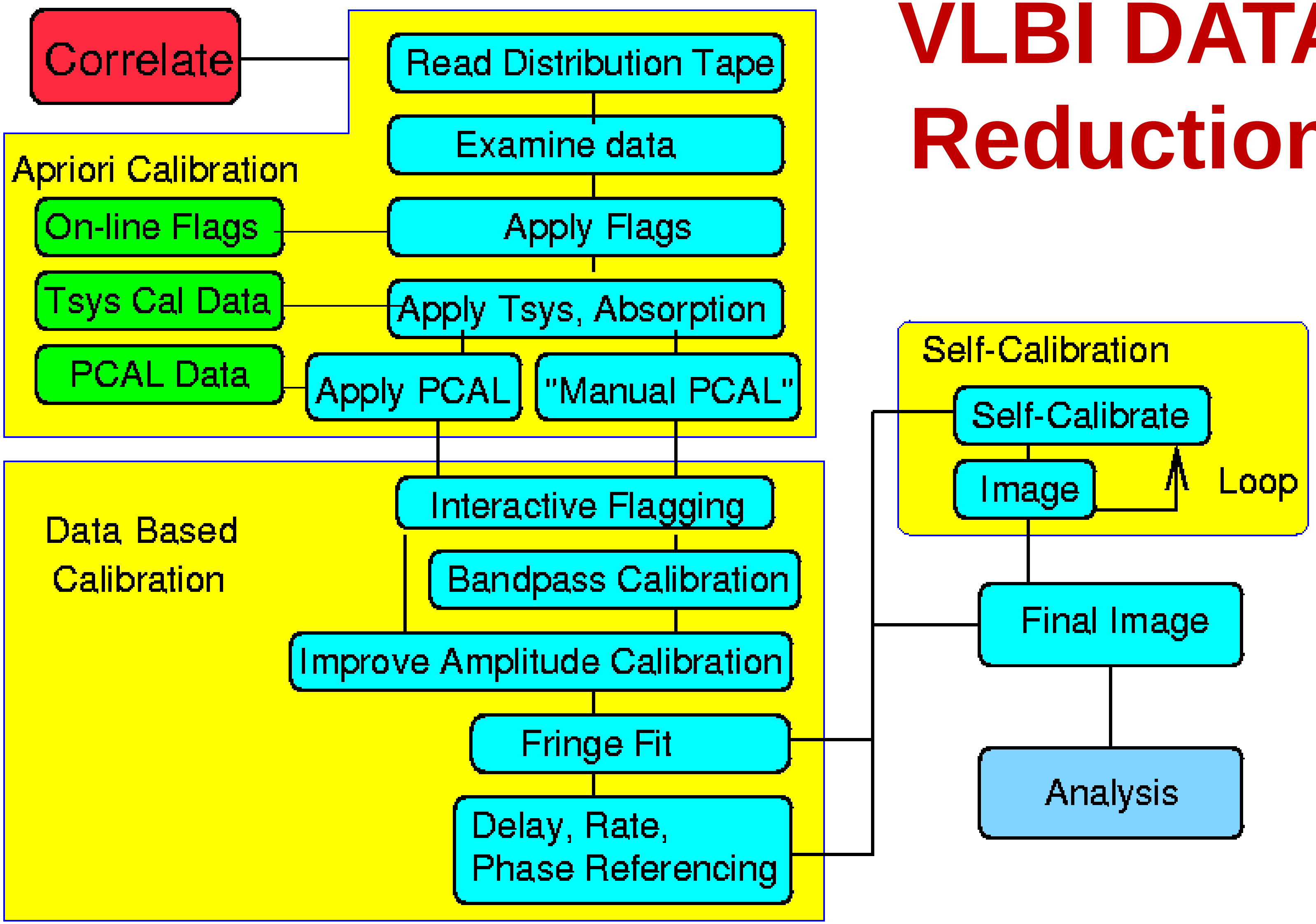
Growth of telecom industry threatening radio astronomy!

L BAND, VLA ARRAY CONFIG "B", 19980701



FREQ(MHz) Note: The 13, −1 values (eg: @1291.25,1308.75,1325, etc.) = sys drop−out errors.

VLBI DATA Reduction



Practical Calibration Considerations

- A priori “calibrations” (provided by the observatory)
  - Antenna positions, earth orientation and rate
  - Clocks, frequency reference
  - Antenna pointing/focus, voltage pattern, gain curve
  - Calibrator coordinates, flux densities, polarization properties
  - T_{sys}, nominal sensitivity
- Absolute *engineering* calibration (dBm, K, Volts)?
  - Very difficult, requires heroic efforts by observatory scientific and engineering staff
  - **Amplitude:** T_{sys}, or switched-power monitoring to enable calibration to nominal K, or Jy with antenna efficiency information
  - Phase: inject phase-cal, water vapor radiometer (ALMA)
  - Traditionally we concentrate instead on ensuring instrumental *stability* on adequate timescales

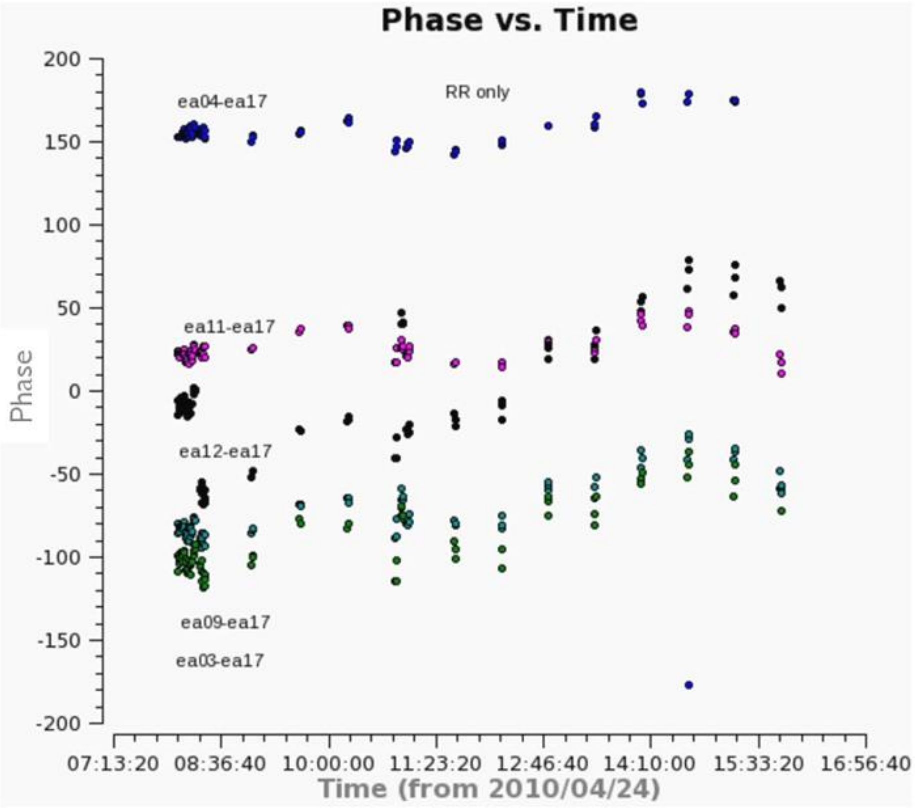
Practical Calibration: Cross Calibration

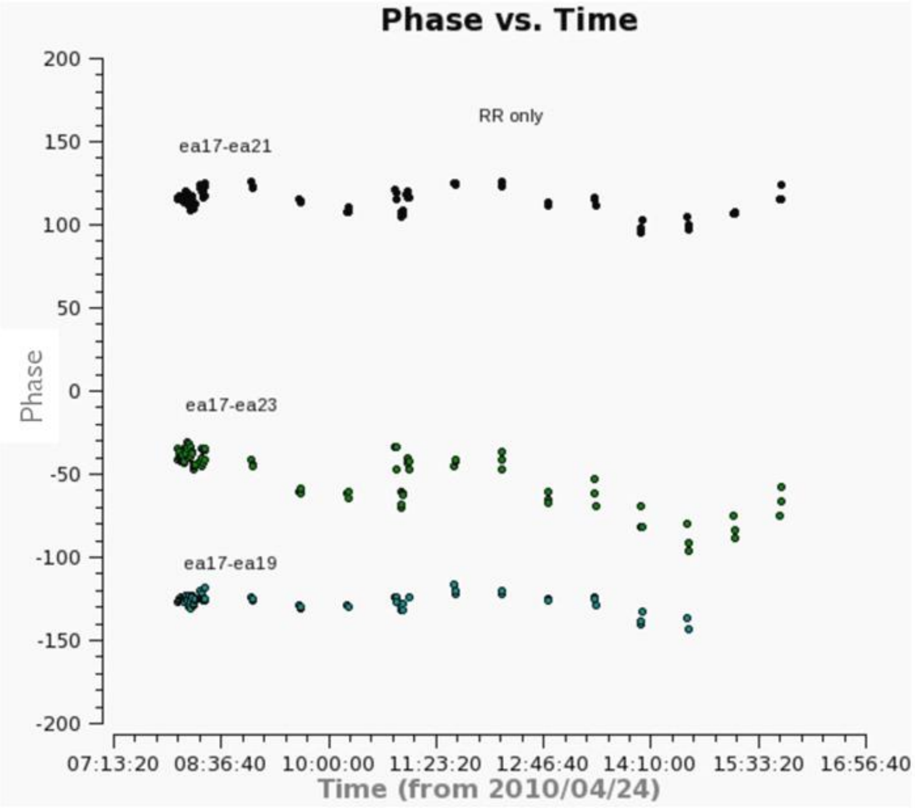
- ***Cross-calibration*** a better choice
  - Observe strong sources – calibrator sources or just calibrators - near the science target whose characteristics, position, flux density, are known!
  - solve for calibration against calibrators and transfer solutions to target observations
  - Choose appropriate calibrators; usually strong point sources because we can easily predict their visibilities: amplitude = constant, phase = 0
    - VLBI: not so easy! most sources somewhat resolved
  - Choose appropriate timescales for calibration (typically minutes; usually longer at low frequencies, shorter at high frequencies)

Antenna-based Cross Calibration

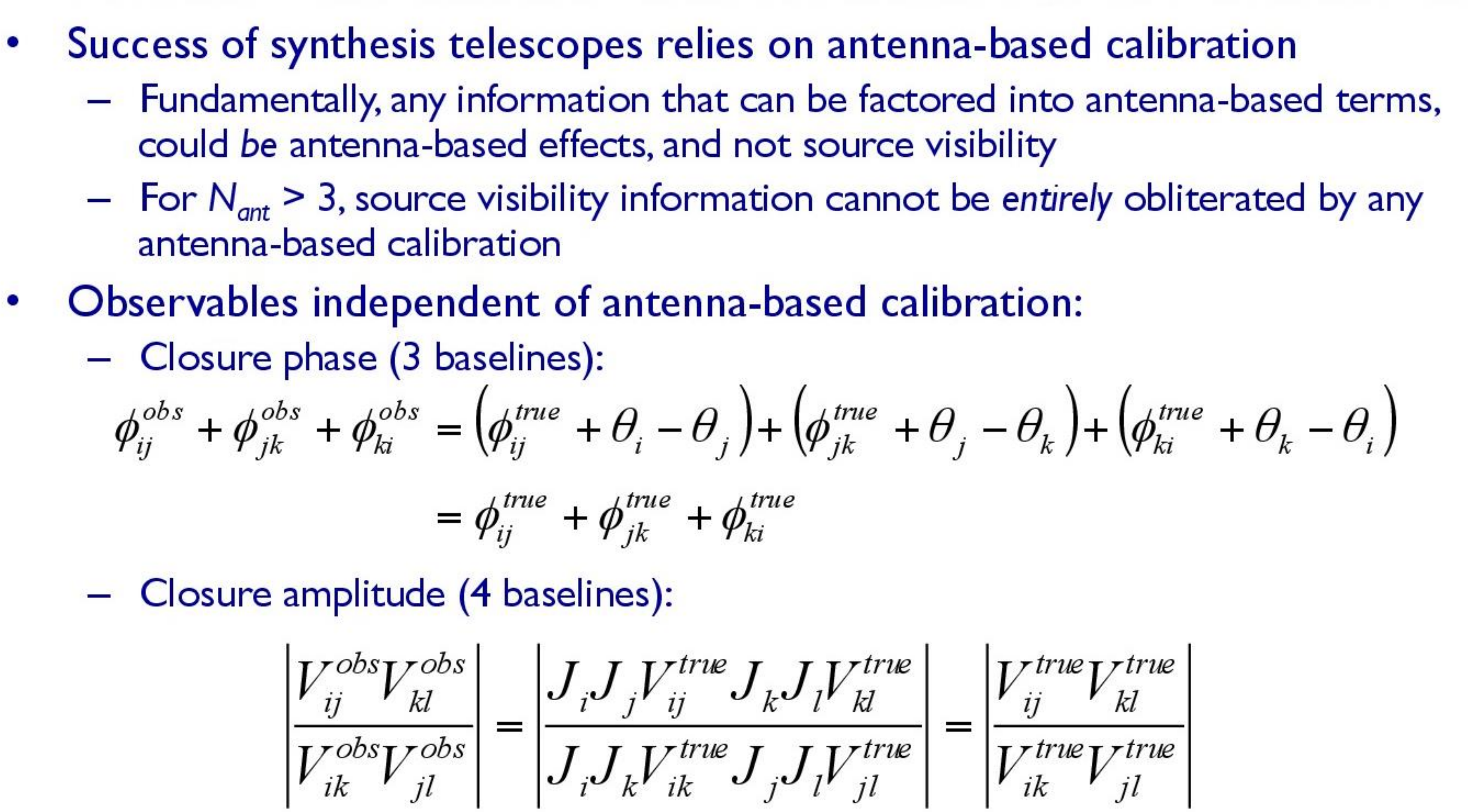
- Measured visibilities are formed from a product of *antenna-based* signals – we can take advantage of this:
- N antennas, there are $N_{\text{baseline}} = N*(N\text{-}1)/2 \sim N^2/2$ baselines.
- Take calibration factor for baseline i,j *to* be $G_{ij,}$ so you need to determine N_{baseline} factors $G_{ij,}$
- If calibration factors into antenna-based factors. so calibration for baseline i,j then $G_{ij,} = G_i \times G_{j,}$ and you need only N factors G_i - much easier if N is large
- Luckily many effects *are* antenna dependent – that is they effect all baselines to any antenna (at some given time) the same way.

Rationale for Antenna-Based Solution





Antenna-based Calibration and Closure



- Success of synthesis telescopes relies on antenna-based calibration
  - Fundamentally, any information that can be factored into antenna-based terms, could *be* antenna-based effects, and not source visibility
  - For $N_{ant} > 3$, source visibility information cannot be *entirely* obliterated by any antenna-based calibration
- Observables independent of antenna-based calibration:
  - Closure phase (3 baselines):

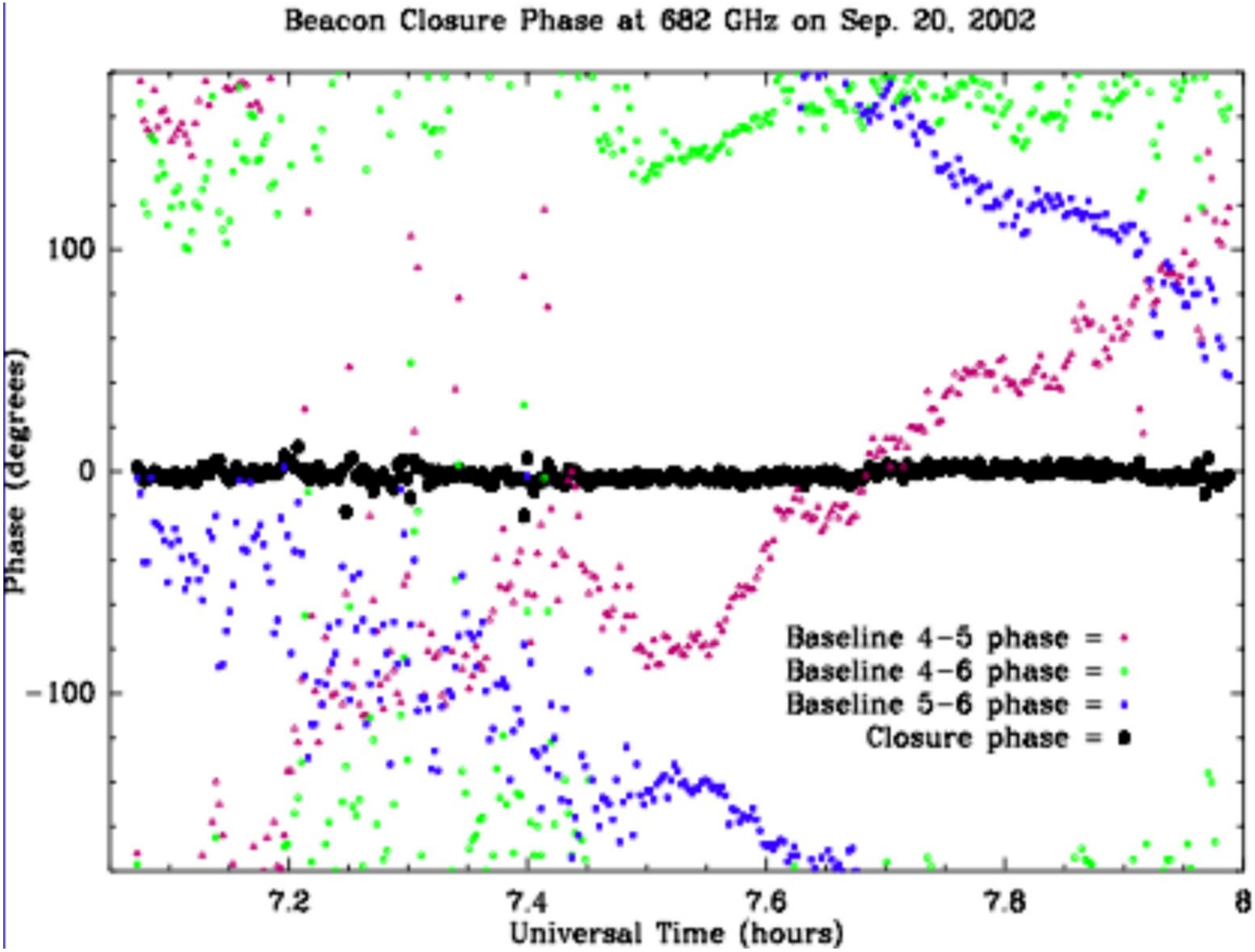
$$\phi_{ij}^{obs} + \phi_{jk}^{obs} + \phi_{ki}^{obs} = \left(\phi_{ij}^{true} + \theta_i - \theta_j\right) + \left(\phi_{jk}^{true} + \theta_j - \theta_k\right) + \left(\phi_{ki}^{true} + \theta_k - \theta_i\right)$$

$$= \phi_{ij}^{true} + \phi_{jk}^{true} + \phi_{ki}^{true}$$

  - Closure amplitude (4 baselines):

$$\left|\frac{V_{ij}^{obs} V_{kl}^{obs}}{V_{ik}^{obs} V_{jl}^{obs}}\right| = \left|\frac{J_i J_j V_{ij}^{true} J_k J_l V_{kl}^{true}}{J_i J_k V_{ik}^{true} J_j J_l V_{jl}^{true}}\right| = \left|\frac{V_{ij}^{true} V_{kl}^{true}}{V_{ik}^{true} V_{jl}^{true}}\right|$$

Closure Phase



Sub-millimetere Array 682 GHz

VLBI Amplitude Calibration

$$S_{cij} = \rho \frac{A}{\eta_s} \sqrt{\frac{T_{si} T_{sj}}{K_i K_j e^{-\tau_i} e^{-\tau_j}}}$$

S_{cij} = Correlated flux density on baseline *i - j*

ρ = Measured (normalized) correlation coefficient (amplitude 0 to 1)

A = Correlator specific scaling factor

η_s = System efficiency including digitization losses

T_s = System temperature

Includes receiver, spillover, atmosphere, blockage

K = Gain in degrees K per Jansky

Includes dependence of antenna gain on elevation

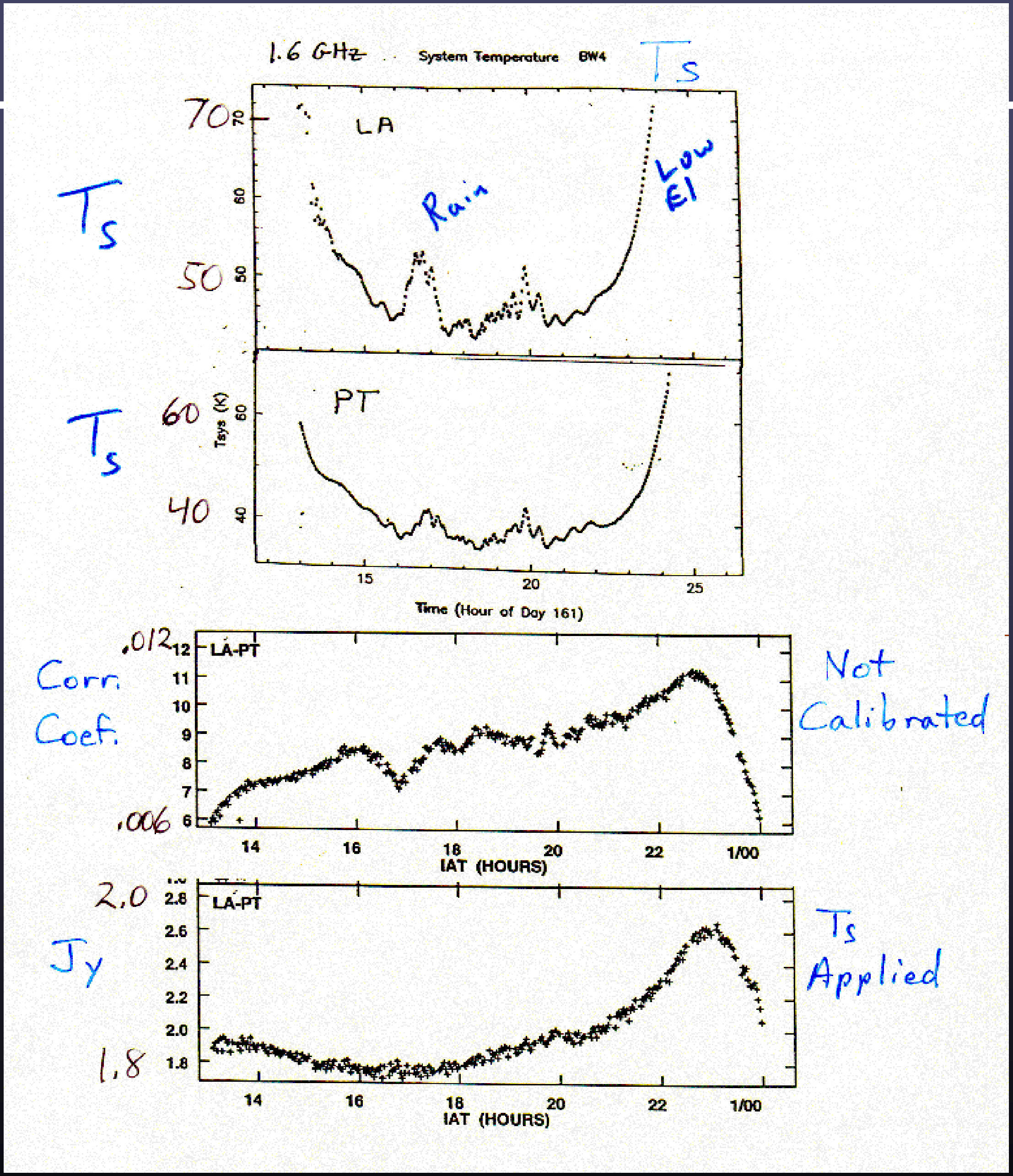
$e^{-\tau}$ = Absorption in atmosphere

Note T_s/K = *SEFD* (System Equivalent Flux Density)

Calibration with Tsys

Example shows removal of effect of increased Tsys due to rain and low elevation

For interferometry, the noise does not correlate, between 2 antennas, so adding more noise decreases any correlation and thus the signal we are interested in



Calibration

The measured visibility *V’* is related to the source visibility *V* as

$$<E_1E^2> = V'(u,v)$$

$$= A'(u,v)\, e^{i[\psi(u,v)]} = g_1g_2\, A(u,v)\, e^{i[\varphi(u,v)+\phi(u,v)]}$$

$$= g_1g_2\, e^{i[\phi(u,v)]} \times V(u,v)$$

where ψ is the measured phase, φ is the true source phase and ϕ is phase shift due to the electronics, atmosphere and ionosphere, where 1, 2 denote the two antennas

Calibration is to determine $g_1g_2\, \mathrm{e}^{\mathrm{i}[\phi(\mathrm{u,v})]}$,where the phase shift is typically antenna based. i.e.

$$\phi(12) = [\phi_e(1) - \phi_e(2)] + [\phi_a(1) - \phi_a(2)] + [\phi_i(1) - \phi_i(2)] \ldots$$

Observe calibrations that are point sources of known flux *S* and known position ($\varphi = 0$), and the measured

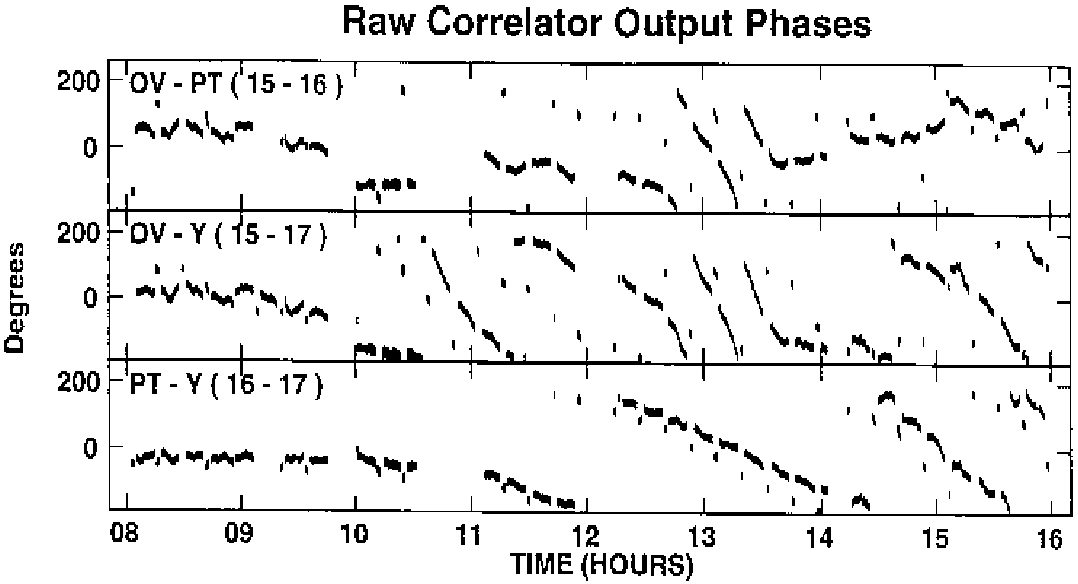
$$V'(u,v)/S = g_1g_2\, \mathrm{e}^{\mathrm{i}[\phi(\mathrm{u,v})]} = G_1G_2^*$$

where the complex *G* represents the amplitude and phase that needs to be removed to yield the true source visibilities

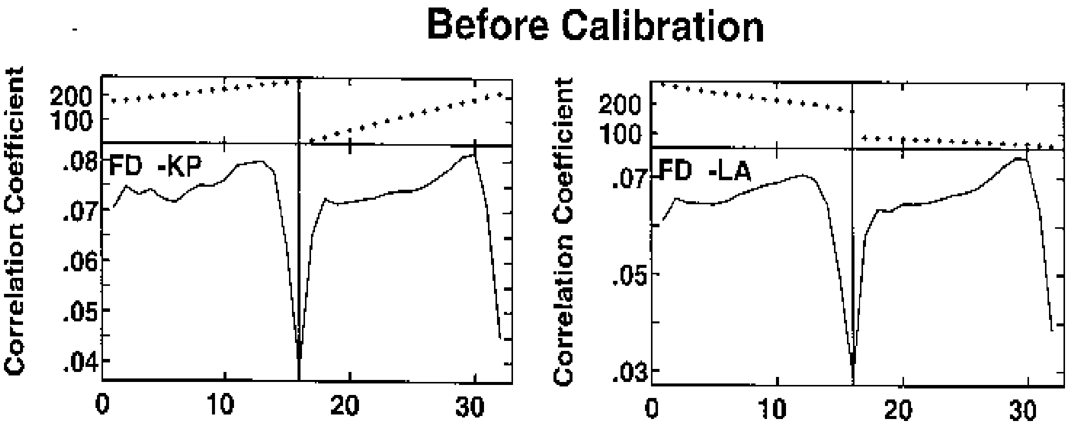
You measure (phase) calibrators regularly throughout the observations to provide solutions (as a function of time) on *N* factors *G* from $N(N-1)/2$ (baseline) measurements. The G(*t*) are then applied to the observations of the source.

Fringe Fitting

- Raw correlator output has phase slopes in time and frequency
  - Slope in time is “fringe rate”
    - Usually from imperfect troposphere or ionosphere model
  - Slope of visibility phase in frequency is “delay”
    - A phase slope because $\phi=\upsilon\tau$
    - Fluctuations worse at low frequency because of ionosphere
    - Troposphere affects all frequencies equally ("nondispersive")
- Fringe fit is self-calibration with first derivatives in time and frequency



S. Doeleman



Channel (corresponds to frequency)

Fringes: Example

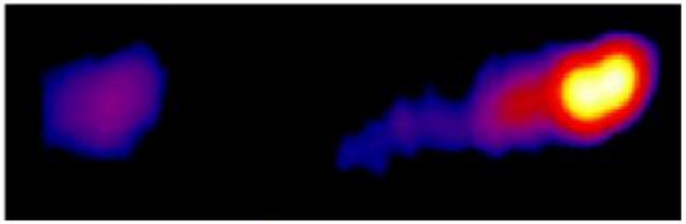
100 000 km from Earth

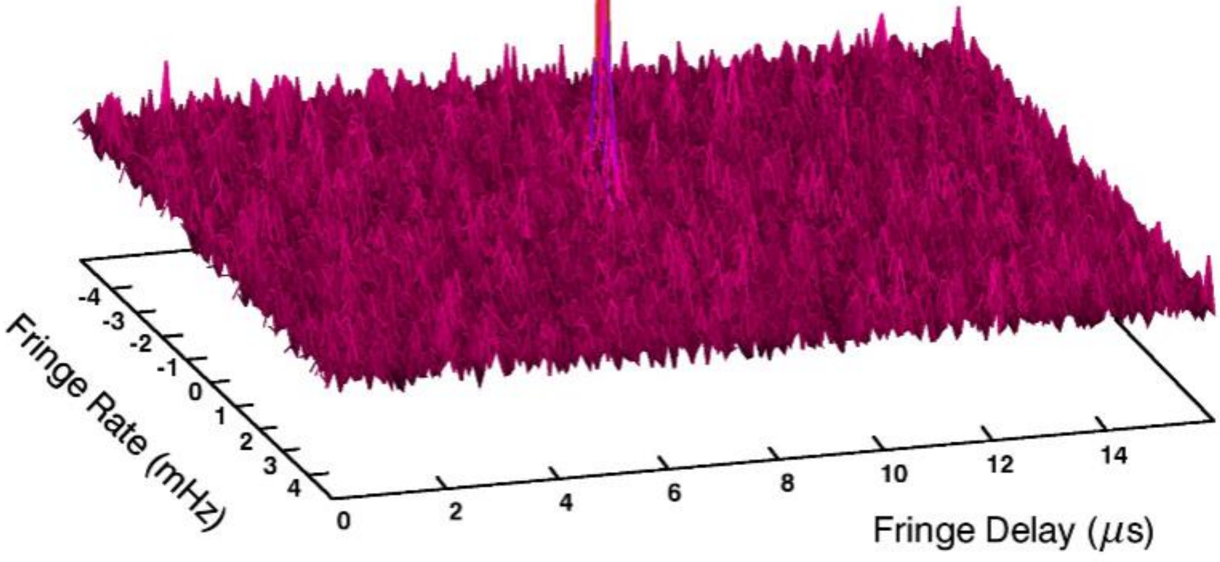
November 15, 2011
Wavelength 18 cm

Baseline projection: 50 Mλ

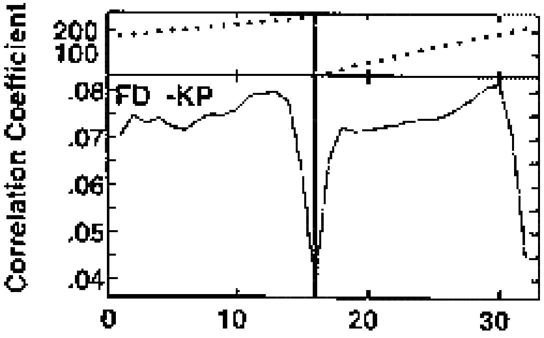
SRT-Effelsberg

Interferometric signal from quasar 0212+735





Raw phase vs. frequency



Why do we need to Fringe Fit?

- Correlator model is good, but not perfect
  - Typically, antenna models and locations are now very good, but...
  - Source positions are imperfect, and can vary with time and frequency
- Atmosphere and ionosphere are time-variable and unpredictable
- Clock information has significant errors at the VLBI level of accuracy

Slide: Olaf Wucknitz

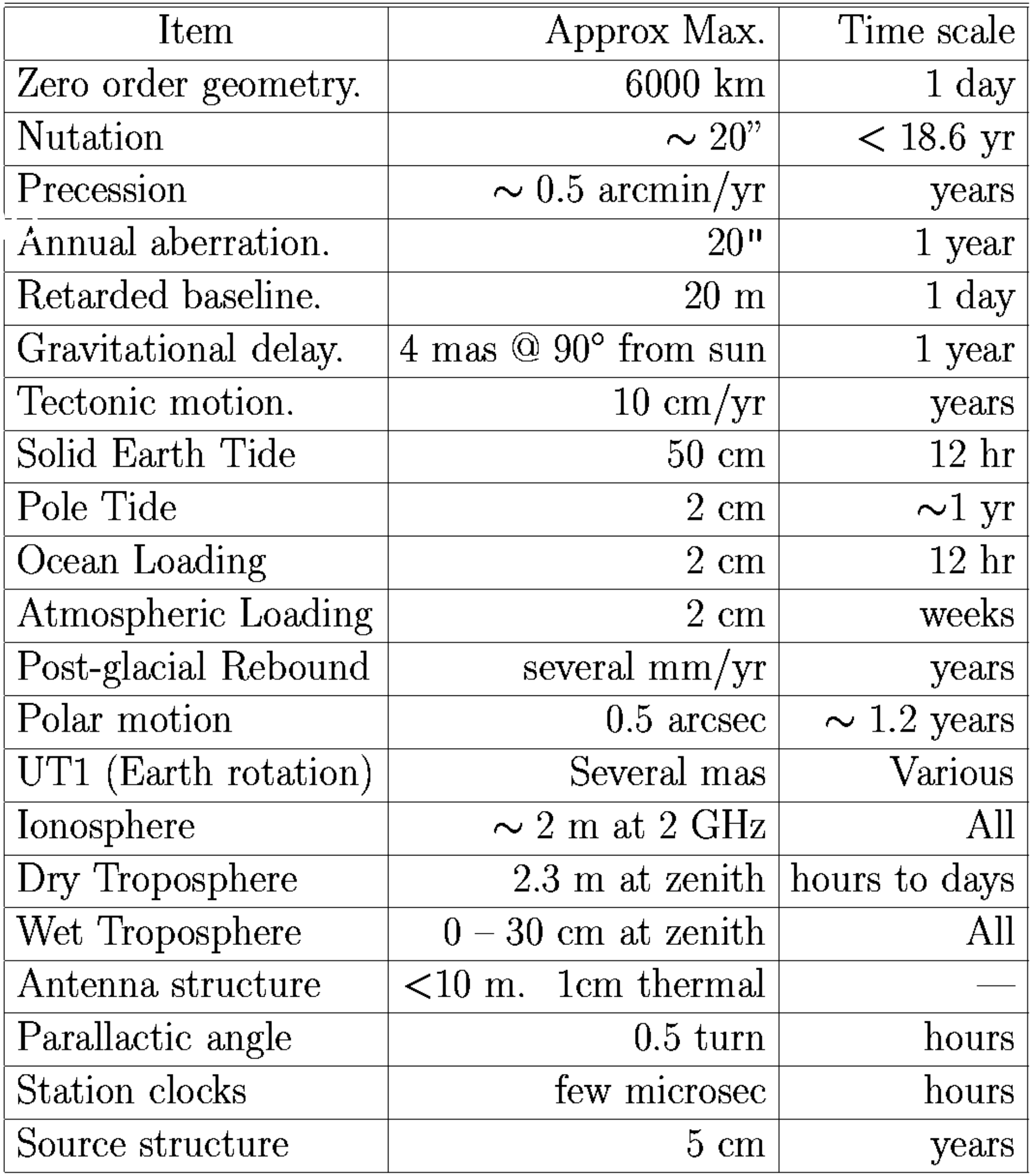
Delay & Rate

- effect of a delay τ
  - ⋆ telescope signal $$V_j(t) = A_j e^{2\pi i\nu(t-\tau_j)}$$
  - ⋆ correlation $$\langle V_1 V_2^\star \rangle = A_1 A_2^\star e^{2\pi i\nu(\tau_2-\tau_1)}$$
  - ⋆ phase $$\boxed{\phi = 2\pi\nu(\tau_2 - \tau_1)}$$
- frequency dependence
  - ⋆ $$\boxed{\frac{\partial\phi}{\partial\nu} = 2\pi\tau}$$ ‘delay’ is frequency-derivative of phase
- phase rate and delay rate
  - ⋆ $$\boxed{\frac{\partial\phi}{\partial t} = 2\pi\nu\frac{\partial\tau}{\partial t}}$$ equiv. Doppler effect, frequency error

Slide: Olaf Wucknitz

THE DELAY MODEL

Adapted from Sovers, Fanselow, and Jacobs (yes that’s *Chris*), 1998 Reviews of Modern Physics



Item	Approx Max.	Time scale
Zero order geometry.	6000 km	1 day
Nutation	~ 20”	< 18.6 yr
Precession	~ 0.5 arcmin/yr	years
Annual aberration.	20"	1 year
Retarded baseline.	20 m	1 day
Gravitational delay.	4 mas @ 90° from sun	1 year
Tectonic motion.	10 cm/yr	years
Solid Earth Tide	50 cm	12 hr
Pole Tide	2 cm	~1 yr
Ocean Loading	2 cm	12 hr
Atmospheric Loading	2 cm	weeks
Post-glacial Rebound	several mm/yr	years
Polar motion	0.5 arcsec	~ 1.2 years
UT1 (Earth rotation)	Several mas	Various
Ionosphere	~ 2 m at 2 GHz	All
Dry Troposphere	2.3 m at zenith	hours to days
Wet Troposphere	0 – 30 cm at zenith	All
Antenna structure	<10 m. 1cm thermal	—
Parallactic angle	0.5 turn	hours
Station clocks	few microsec	hours
Source structure	5 cm	years

Phase Referencing

- One kind of antenna-based cross-calibration
- Observe a calibrator source nearby your target
- Calibrator source needs to have accurately known position and ideally be point-like
- Derive calibration: complex gains (amplitude and phase), rates, delays from calibrator
- Transfer them to target

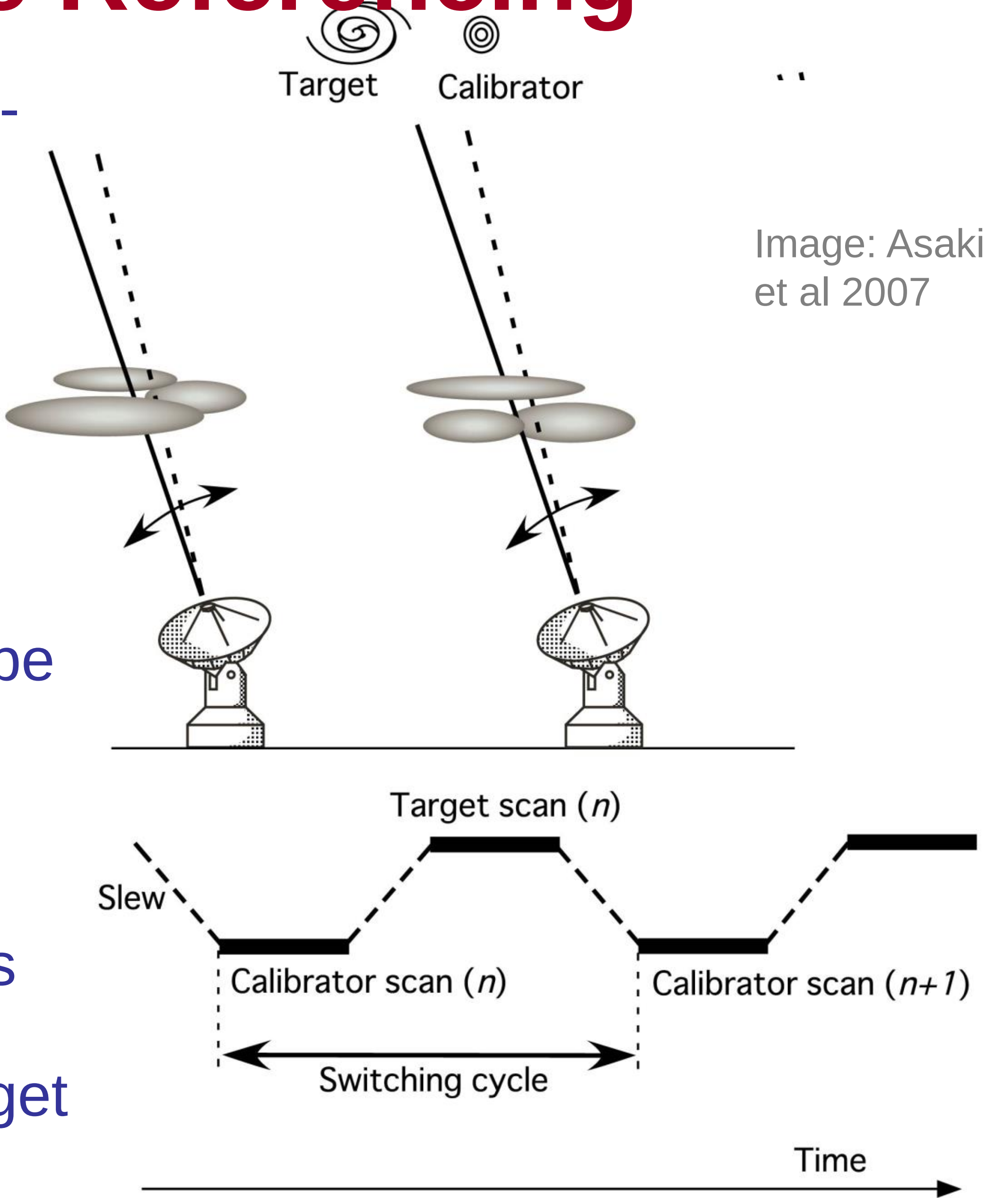
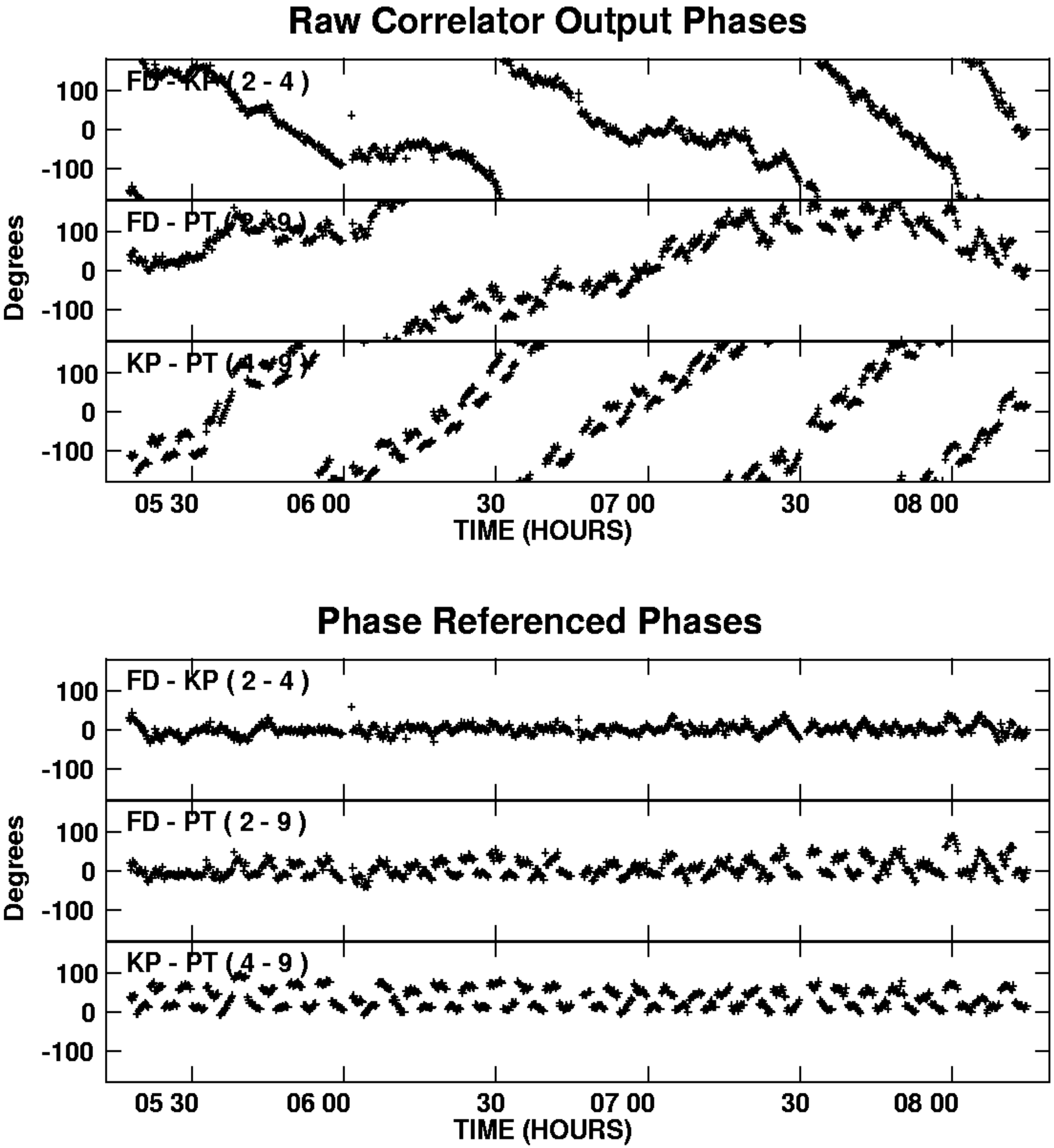


Image: Asaki et al 2007

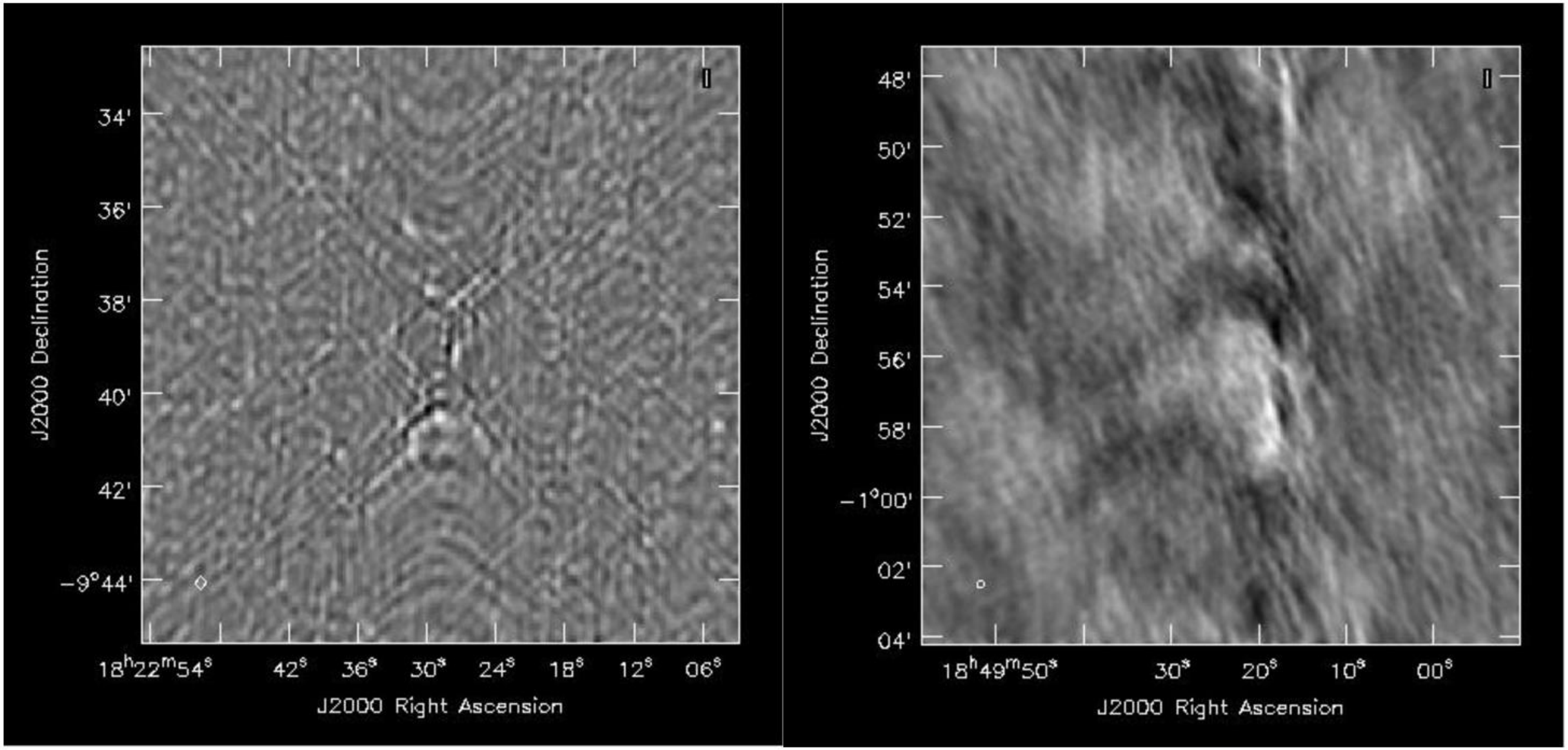
EXAMPLE OF REFERENCED PHASES

- 6 min cycle – 3 min on each source
- Visibility phases of one source were self-calibrated (so after calibration, phases are near zero)
- Phases of the visibilities of the other source phase-shifted by same amount



Slide: Lo & Cornwell

Effect of Calibration in Images

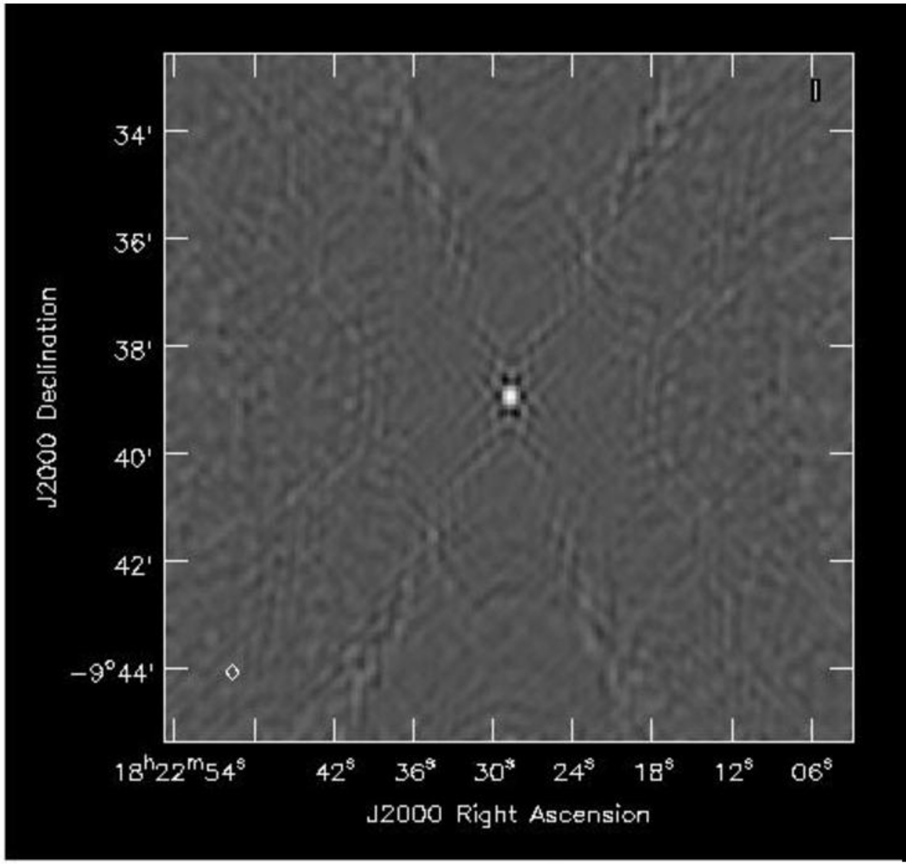


J1822-0938
(calibrator)

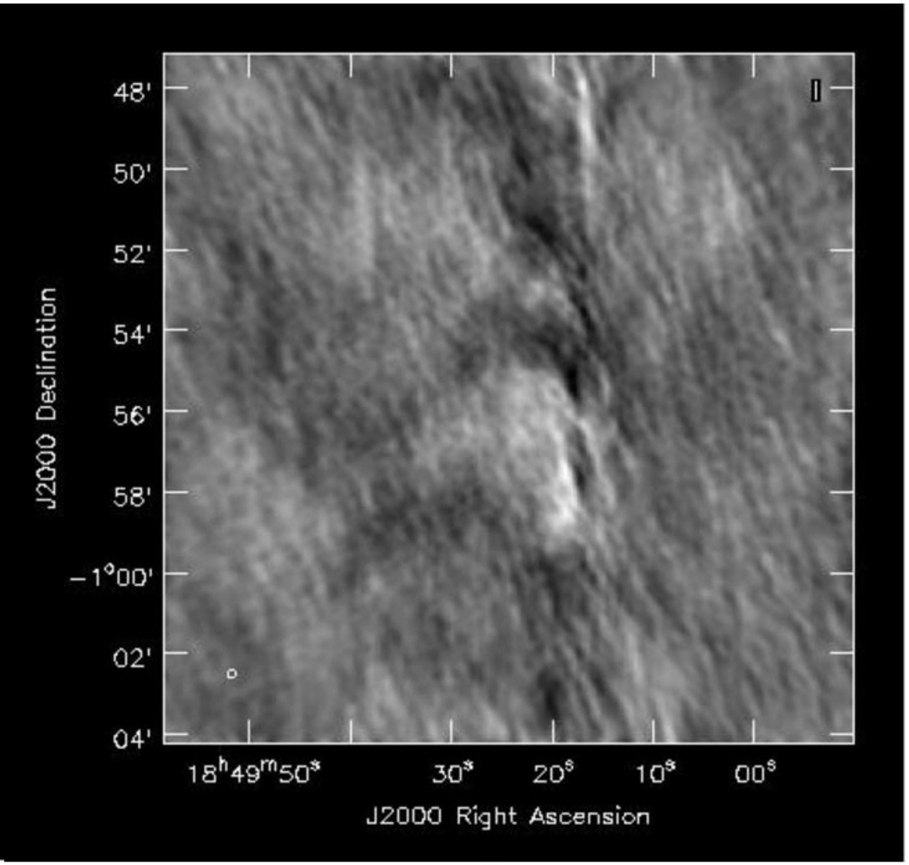
3C391
(science)

Uncalibrated images (VLA) of calibrator J1822-0938 and target 3C391

Effect of Calibration in Images



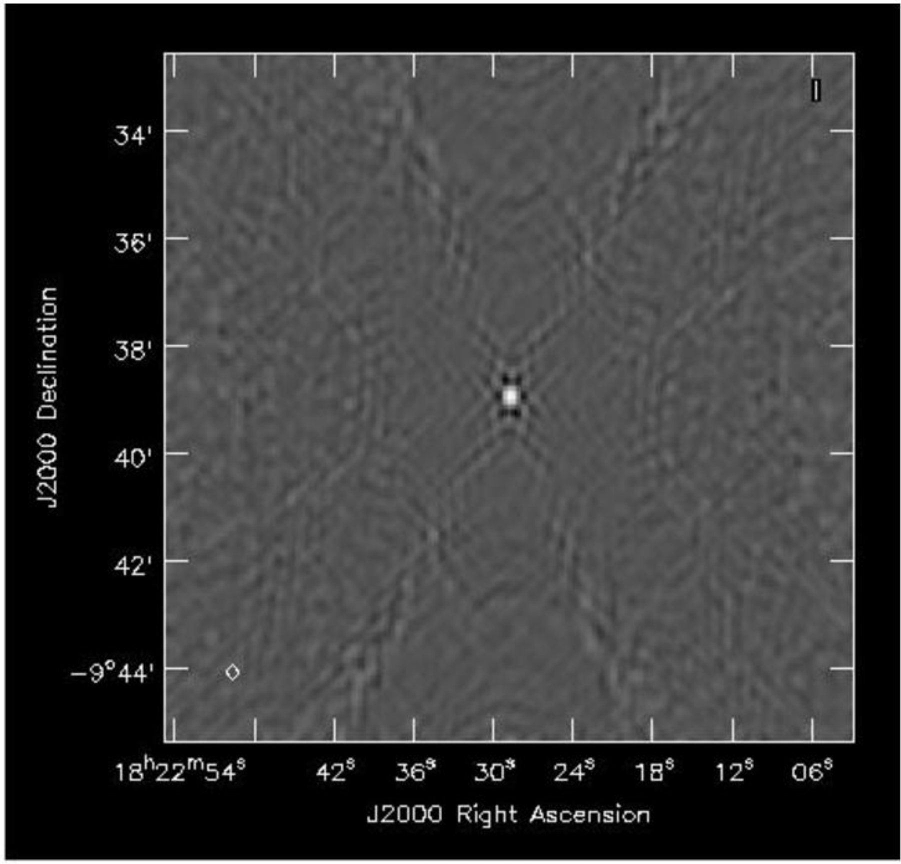
J1822-0938
(calibrator)



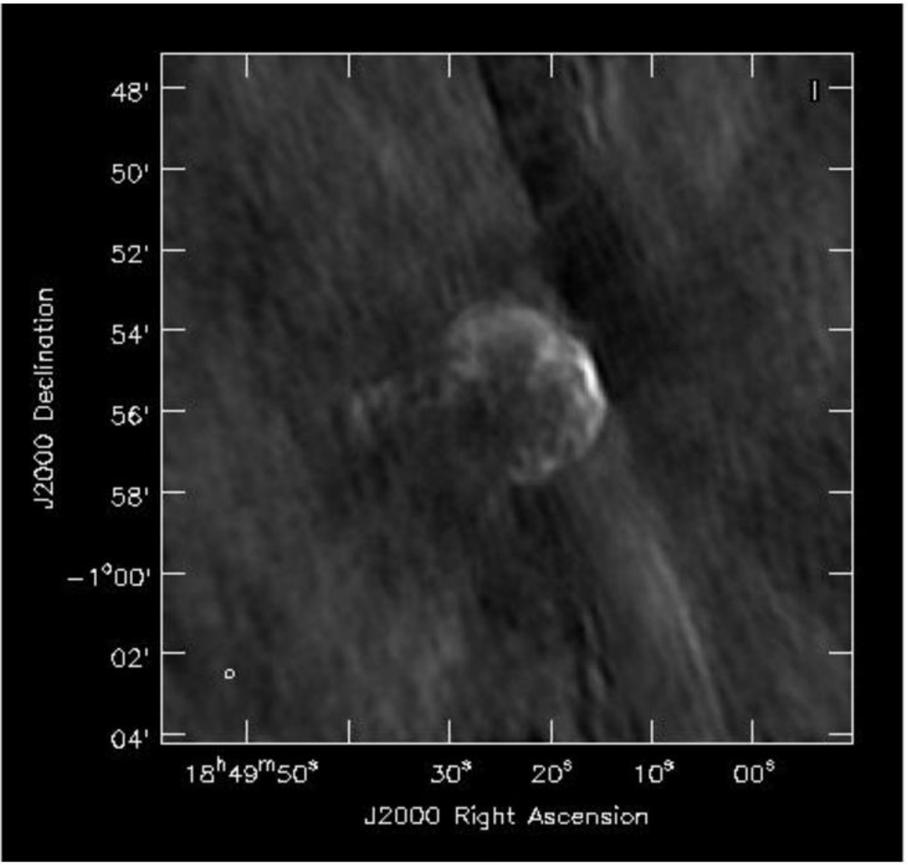
3C391
(science)

Calibrate J1822-0938 (calibrator)

Effect of Calibration in Images



J1822-0938
(calibrator)



3C391
(science)

Transfer calibration solutions to target, 3C391

Stages of a VLBI project

1. Formulate observational science question(s) you wish to investigate.
2. Consider practical observational details:

    -- desired angular resolution, field of view, image sensitivity, observing frequencies, spectral resolution, polarization, temporal coverage

    -- select an appropriate telescope/array
3. Submit an observing proposal
4. Construct an observing schedule file
5. Download, reduce, and analyze the data
6. Publish your results
7. Book your ticket to the Nobel ceremoniy in Stockholm

Slide from Matt Lister,
2012 Purdue U., USA

Summary

Determining calibrations is crucial for getting source properties – you can't have one without the other

Data examination and editing part of the calibration process

Calibration is dominated by antenna-based effects

- permits efficient, accurate and defensible separation of calibration effects from astronomical information (satisfies closure)

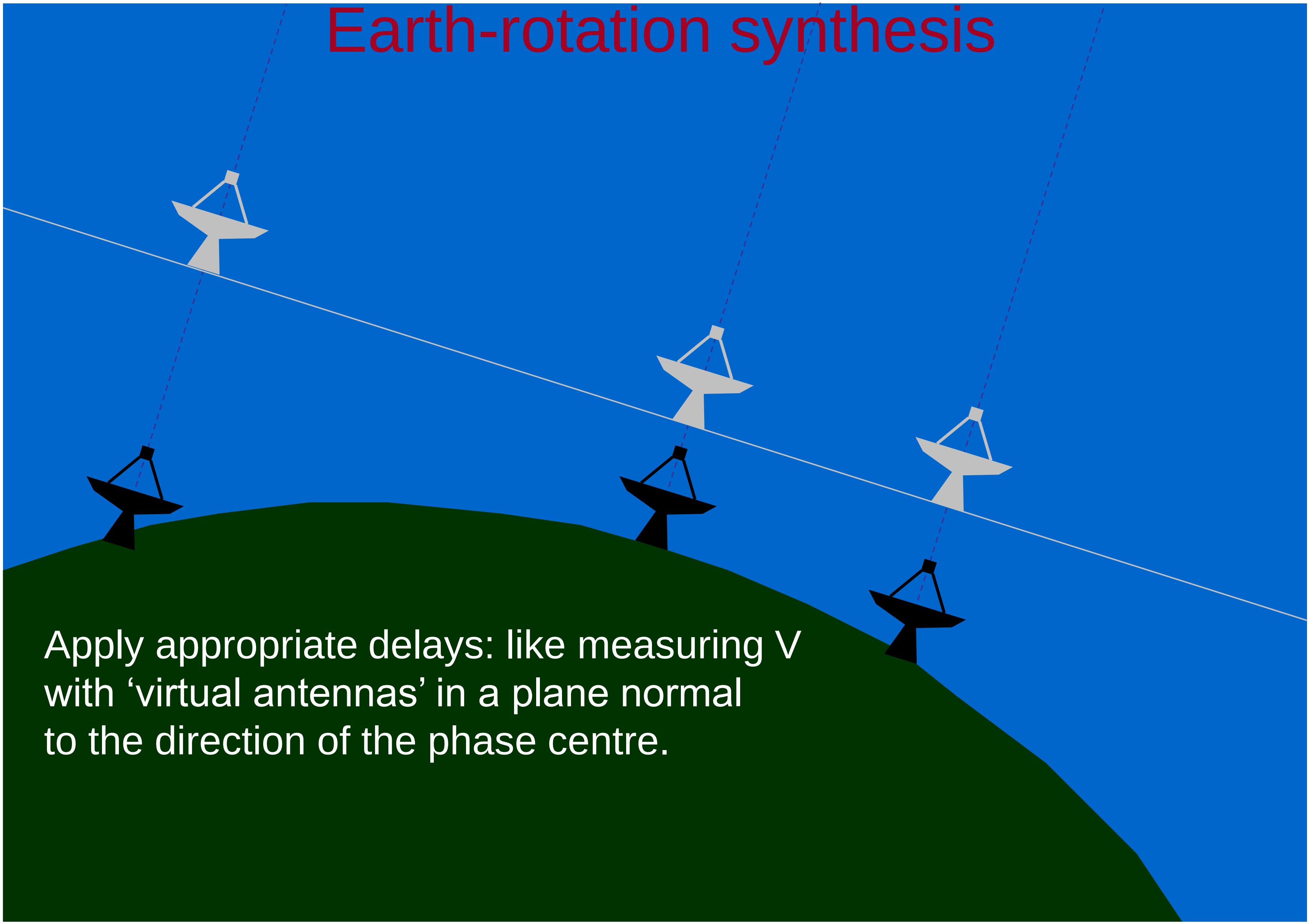
Full calibration formalism is complicated, but its modular

Calibration (including editing) is an iterative procedure: improve estimates of various quantities in turn

Point (unresolved) sources are the best calibrators

Observe calibrators according to the calibration component requirements

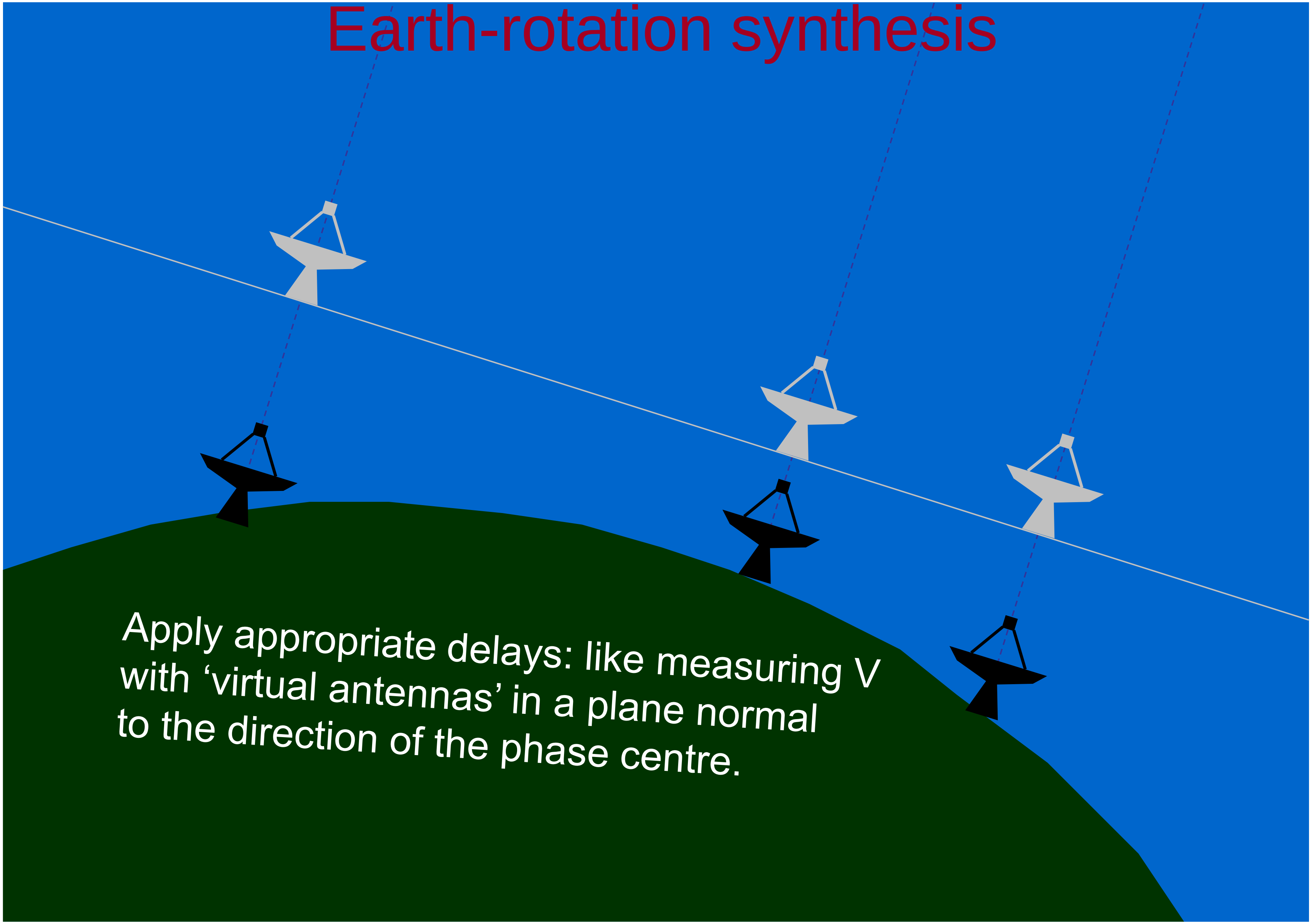
Earth-rotation synthesis



Apply appropriate delays: like measuring V with ‘virtual antennas’ in a plane normal to the direction of the phase centre.

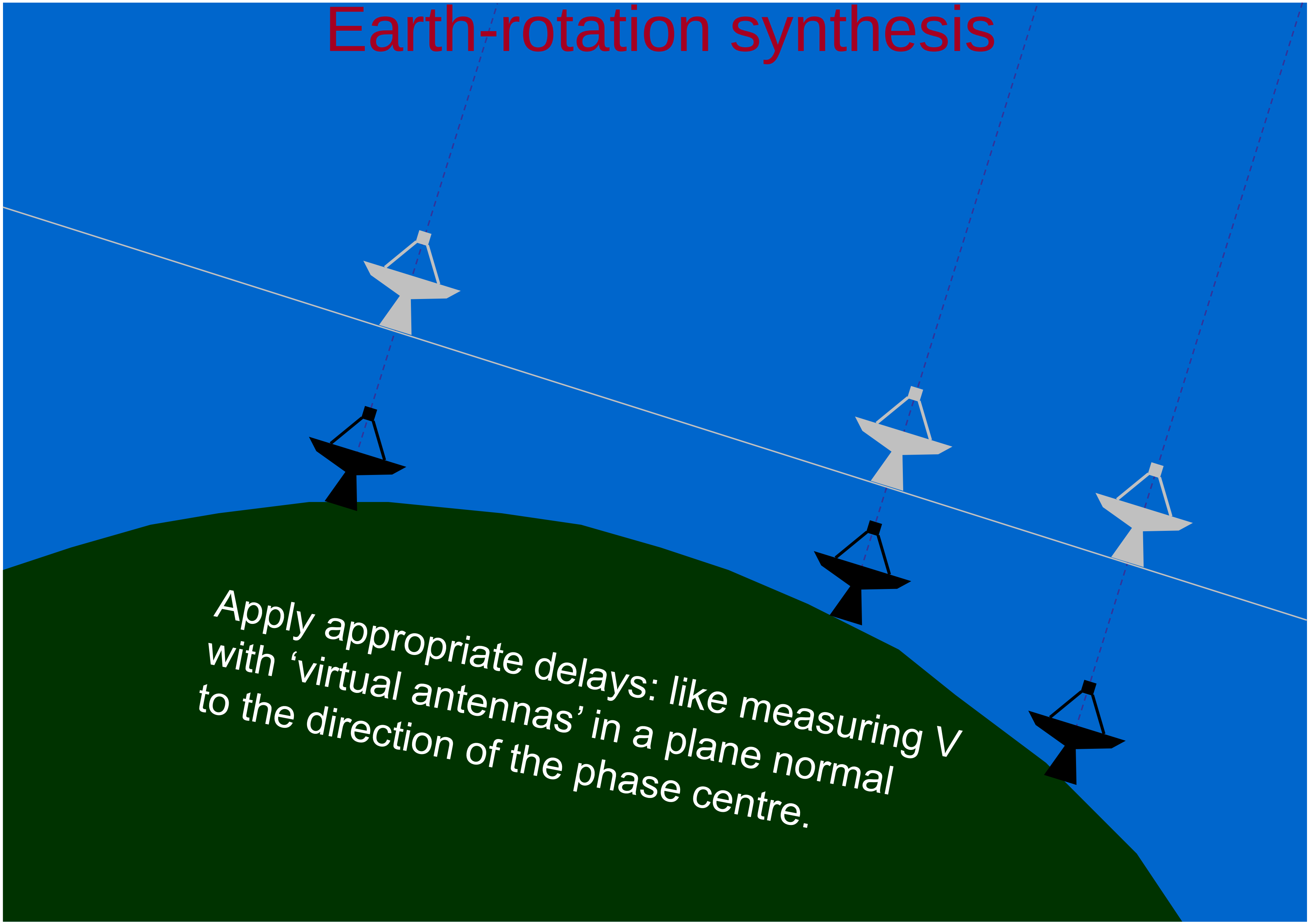
Slide: Ian Stewart

Earth-rotation synthesis



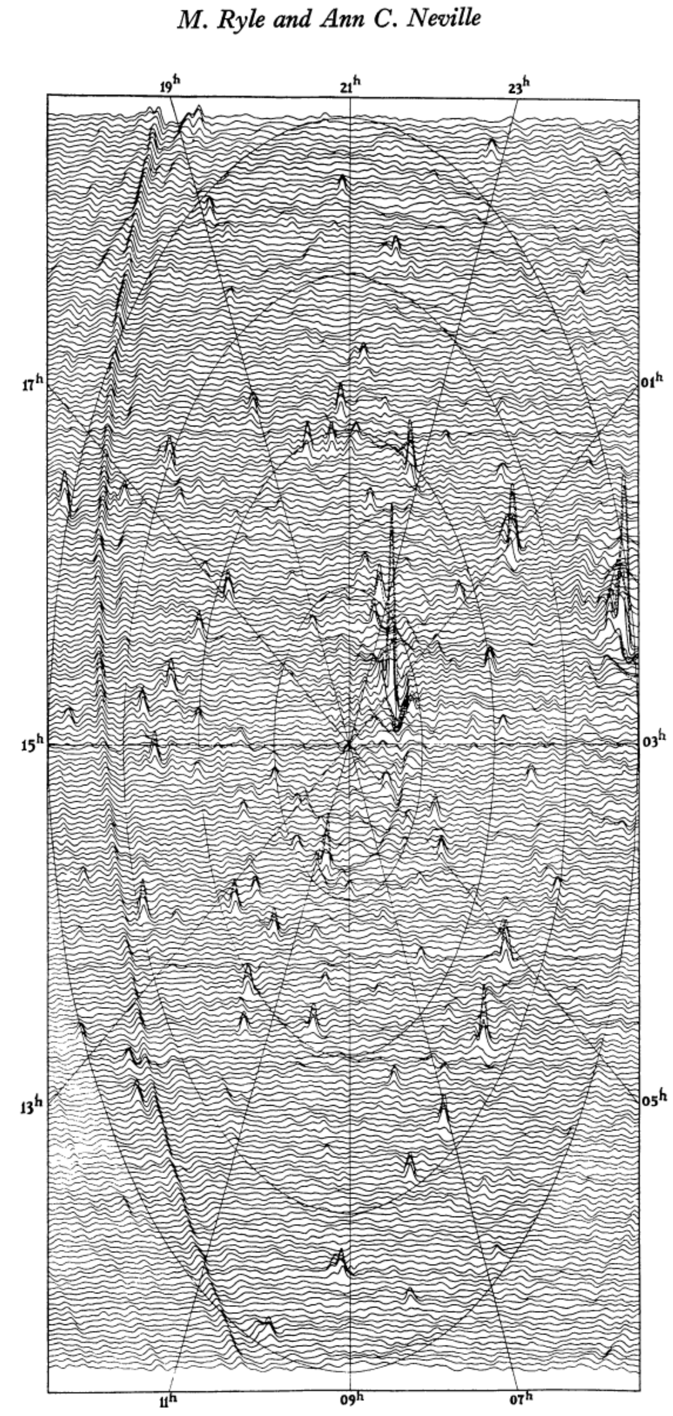
Apply appropriate delays: like measuring V with ‘virtual antennas’ in a plane normal to the direction of the phase centre.

Earth-rotation synthesis



Apply appropriate delays: like measuring V with ‘virtual antennas’ in a plane normal to the direction of the phase centre.

First Cambridge Earth Rotation Synthesis Image



- Ryle & Neville, MNRAS 1962
- June 1961
- North pole survey
- 4C aerials
- 178 MHz
- 7 years after Christiansen
- Similar results now being obtained by LOFAR & MWA!

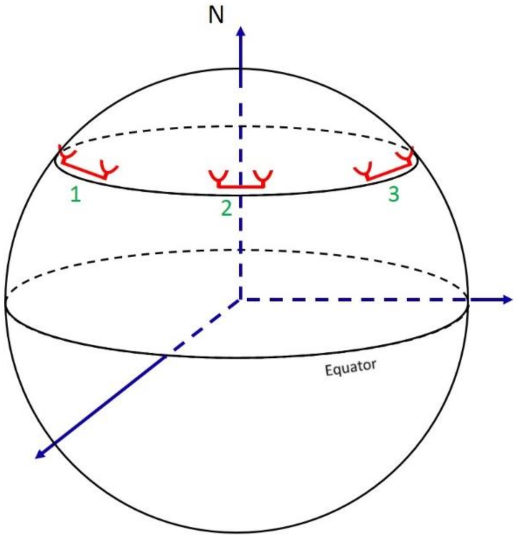
Slide : Ekers “The Development of Aperture Synthesis”

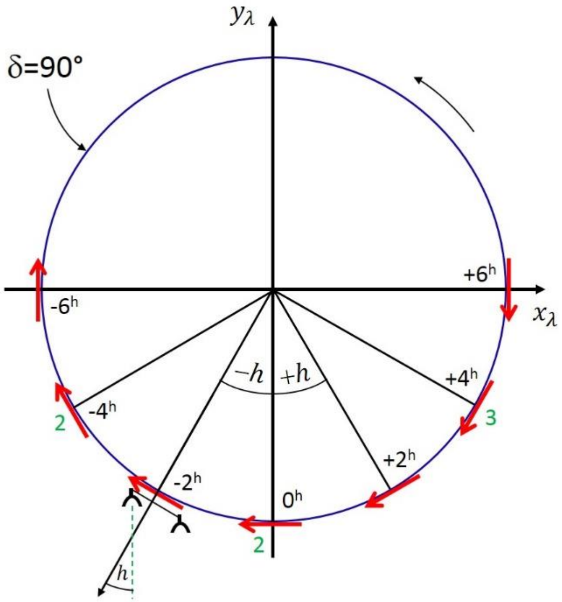
Earth-rotation synthesis

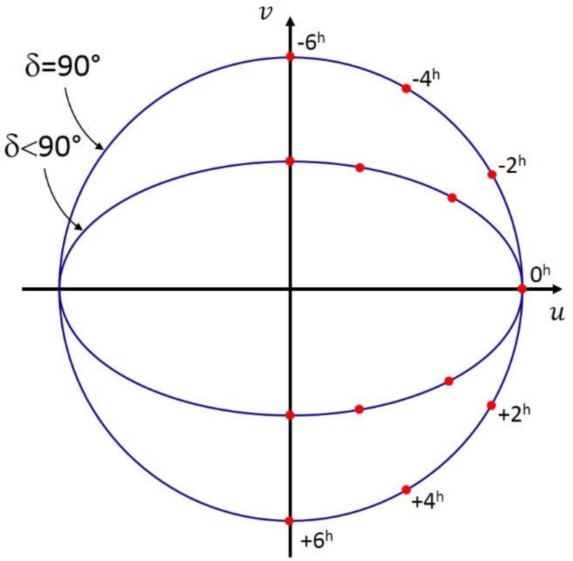
need good sampling of *(u,v)* plane with many (non-redundant) baselines

earth-rotation synthesis helps a lot

due to Sir Martin Ryle (Nobel Prize 1974 with A. Hewish)







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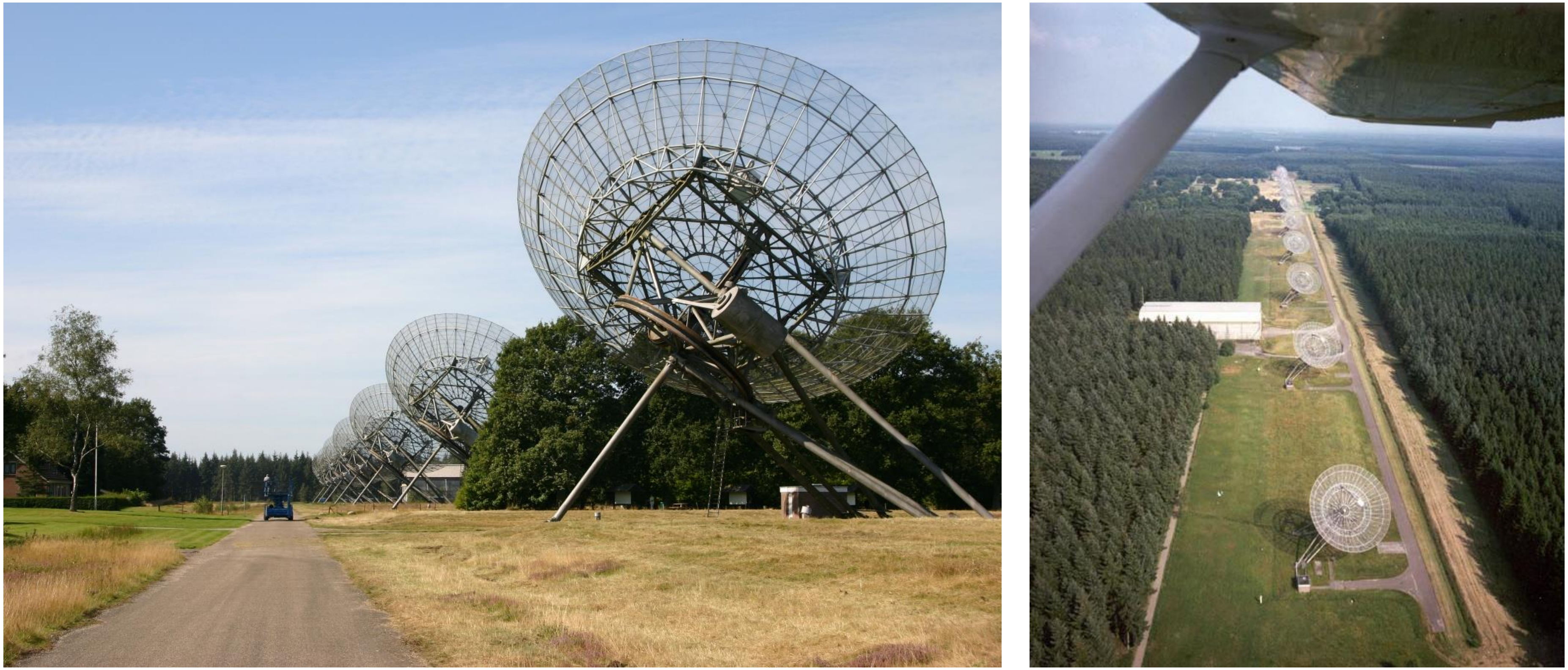
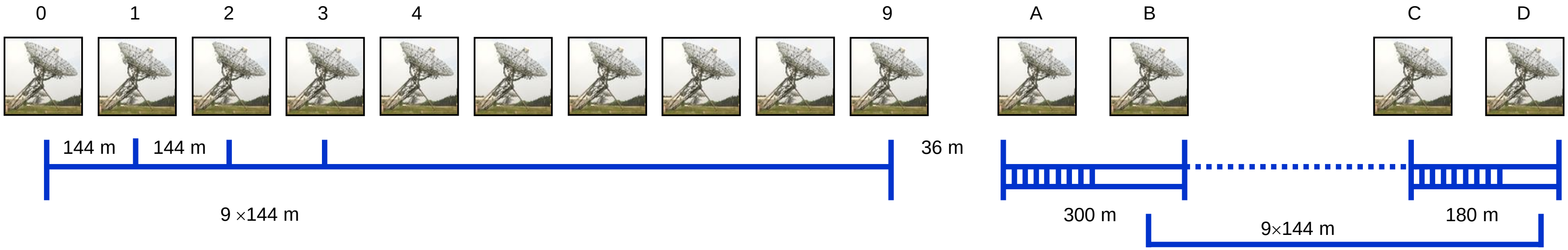
east-west array: e.g. WSRT





0 1 2 3 4 9 A B C D

144 m 144 m

9 ×144 m

36 m

300 m

9×144 m

180 m

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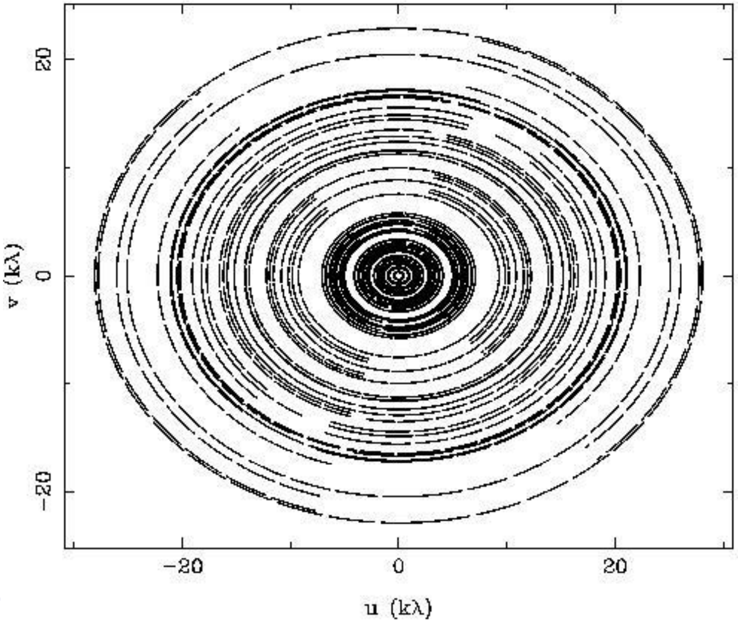
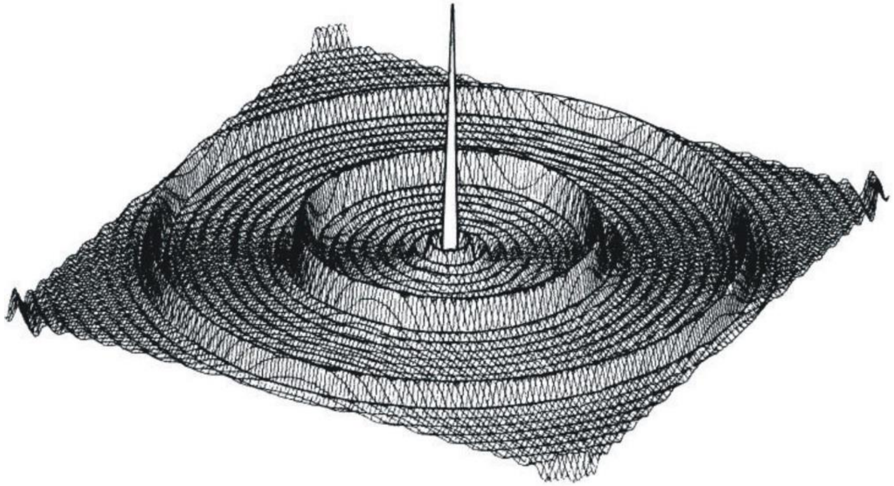
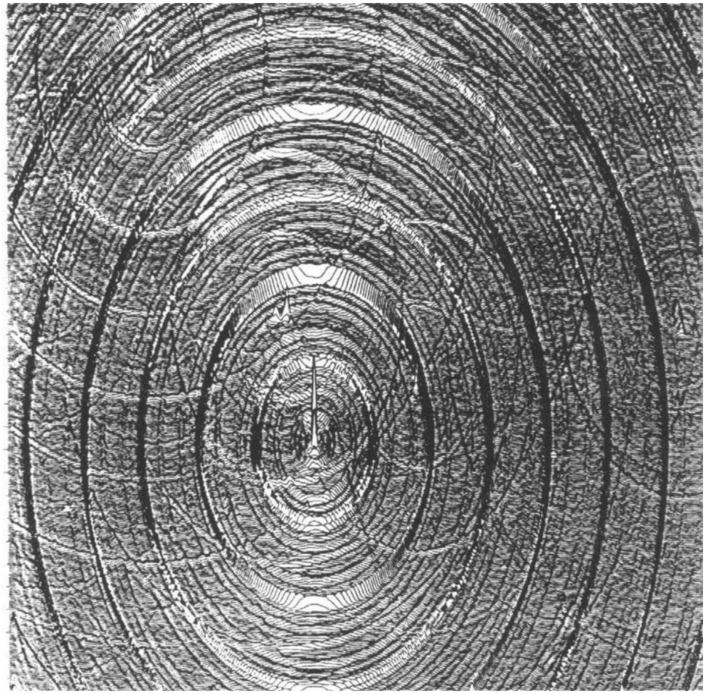
earth rotation produces a good transfer function: (*u*,*v*) tracks

e.g. pure east-west interferometer



FT → antenna diagramme („dirty beam“)





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Stages of a VLBI project

1. Formulate observational science question(s) you wish to investigate.
2. Consider practical observational details:

   -- desired angular resolution, field of view, image sensitivity, observing frequencies, spectral resolution, polarization, temporal coverage

   -- select an appropriate telescope/array
3. Submit an observing proposal
4. Construct an observing schedule file
5. Download, reduce, and analyze the data
6. Publish your results
7. Book your ticket to the Nobel ceremoniy in Stockholm

Slide from Matt Lister,
2012 Purdue U., USA

Reference Antenna

- Since "antenna-based" phase solution is derived from *differences* in antenna-phase since we do not measure phase absolutely
  - relative astrometry
- Phase solutions are typically referred to one specific antenna, the reference antenna ("refant"), which is assumed to have constant phase of zero in both polarizations
  - Typically near the array center
  - The refant's phase variation is distributed to all the other antennas' solutions
  - Adequate time-sampling to ensure reliable interpolation of phase without ambiguity (c.f. arbitrary phase offsets between solutions)
  - Assumes a stable cross-had phase offset (which must be calibrated)
- Problems:
  - A single good refant is not always available over the whole observation (flagging, source above horizon in VLBI)
  - Cross-hand phase difference of refant may not actually be stable ...

Corrected Visibility

- Visibility…

$$V_{ij}^{obs} = J_i J_j^* V_{ij}^{true} \quad \rightarrow \quad V_{ij}^{cor} = J_i^{-1} J_j^{*-1} V_{ij}^{obs}$$

- …and weights!

$$w_{ij}^{cor} = w_{ij}^{obs} \left|J_i\right|^2 \left|J_j\right|^2 = \frac{\left|J_i\right|^2 \left|J_j\right|^2}{\sigma_{ij}^2}$$

  - (calibrate the sigmas)

Data Examination and Editing

- After observation, initial data examination and editing very important
  - Will observations meet goals for calibration and science requirements?
- What to edit:
  - Some real-time flagging occurred during observation (antennas off-source, LO out-of-lock, etc.). Any such bad data left over? (check operator's logs)
  - Any persistently 'dead' antennas (check operator's logs)
  - Periods of poor weather? (check operator's log)
  - Any antennas shadowing others? Edit such data.
  - Amplitude and phase should be continuously varying—edit outliers
  - Radio Frequency Interference (RFI)?
- Caution:
  - Be careful editing noise-dominated data (noise bias).
  - Be conservative: those antennas/timeranges which are bad on calibrators are probably bad on weak target sources—edit them
  - Distinguish between bad (hopeless) data and poorly-calibrated data. E.g., some antennas may have significantly different amplitude response which may not be fatal—it may only need to be calibrated
  - Choose reference antenna wisely (ever-present, stable response)
- Increasing data volumes increasingly demand automated editing algorithms

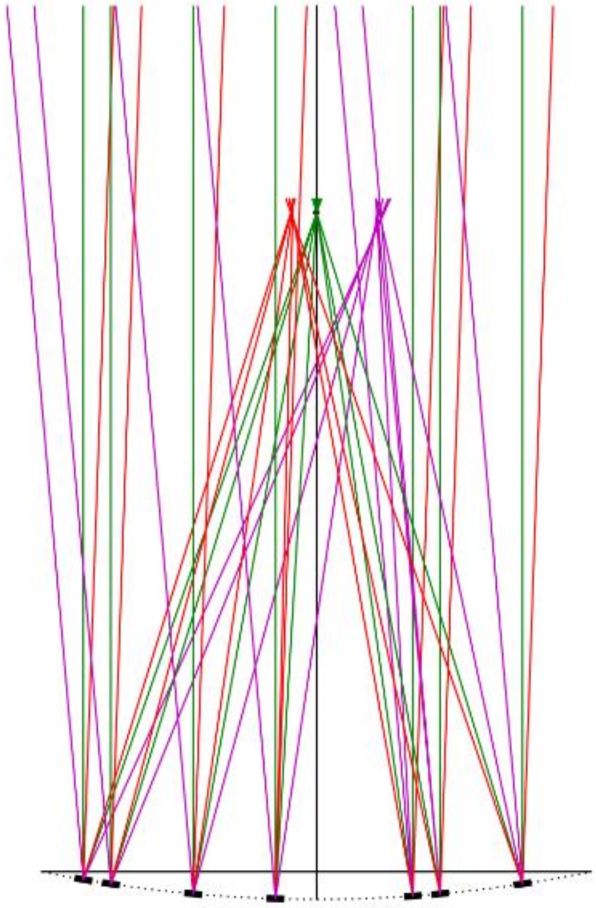
Overview

- Solve for calibration from the visibilities for calibrator sources
- calibration = complex gains = G_i
- Need a model for calibrators: very few compact sources have stable flux densities, so flux density calibrators are often resolved.
- Flux density calibrator (3C286 in this example):
  - We know the complete model (source structure & flux density) so we can solve for complex gains directly
- Secondary ("phase") calibrators:
  - chosen to be near the source.
  - can usually find an unresolved source,
  - but: flux density is almost always variable. However the variability is usually slow, so you can assume the flux density stays constant during observations.
  - so we know source structure but not flux density
  - **Assume** a flux density (1.0 Jy) and solve for gains → gets the gains but with an unknown scaling factor.
  - Scale the gains for secondary calibrator by assuming mean gain of whole array is the same as it is for flux density calibrator.
- Finally, you interpolate as needed for your science targets

Evaluating Calibration Performance

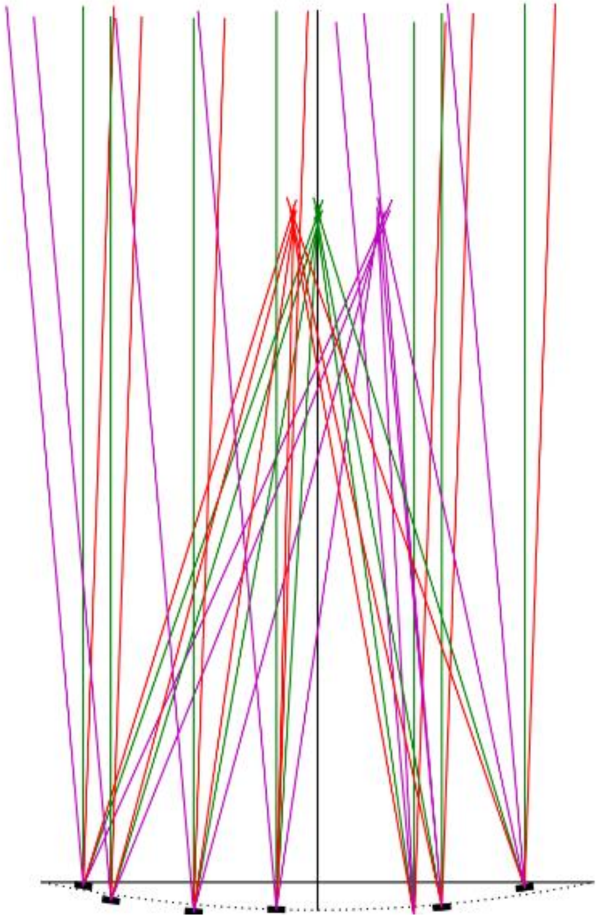
- Are solutions continuous?
  - Noise-like solutions are probably just that—noise
  - Discontinuities indicate instrumental glitches
  - Any additional editing required?
- Are calibrator data fully described by antenna-based effects?
  - Phase and amplitude *closure errors* are the baseline-based residuals
  - Are calibrators sufficiently point-like? If not, self-calibrate: model calibrator visibilities (by imaging, deconvolving and transforming) and re-solve for calibration; iterate to isolate source structure from calibration components
    - John McKean's lecture: "Advanced Calibration" (Tues)
- Any evidence of unsampled variation? Is interpolation of solutions appropriate?
  - Reduce calibration timescale, if SNR permits
- Josh Marvil's lecture: (Weds.)

Undistorted Filled Aperture



- new

Distorted Unfilled Aperture



- Each unfilled aperture segment antenna) has its own distinct properties that uniformly affect all correlations formed with other segments
  - E.g., unmodelled location and electronic path-length errors (delay errors)
  - Complex gain
- Explicit formation and fine sampling of antenna-pair cross-products provides post-observation (“pre-focus”) opportunity to correct errors --- to calibrate