Radio Astronomy with a Satellite Dish

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1 Theory

1.1 Radio Waves

Radio waves are electromagnetic waves having wavelengths greater than a centimetre (Fig. 1). For example, commercial FM radio operates in the frequency range from 88 to 108 MHz, corresponding to a wavelength of about 3 metres. Cellphones operate at a frequency ν of about 900 MHz, i.e. a wavelength λ of 33 cm.

The microwave band is the short wavelength part of the radio band and covers 1 to 30 cm wavelength. Microwave ovens operate at 12 cm (2.4 GHz). DSTV satellites transmit at 2.5 cm (12 GHz). DSTV dishes have a smooth solid surface in order to reflect incoming radio waves with high efficiency. Satellite dishes working at the longer wavelength of 8 cm (3.8 GHz) can get away with a rougher mesh surface and still have acceptable efficiency.

1.2 Radio Telescope Antennas

A "classic" radio telescope for use in the microwave band has a circular parabolic reflector with a feed horn at the focus to collect the incoming microwaves and pass them to transistor amplifiers in the receiver. A DSTV satellite dish also works in this way. It can be used as a mini-radio telescope by replacing the DSTV decoder with a radiometer for measuring the signal strength.

To understand how a reflector antenna responds to radiation coming from different angles, consider what happens when a plane wave of wavelength λ arrives at a circular aperture of diameter D (Fig. 2). Constructive and destructive interference produces a circularly symmetric diffraction pattern, with a central maximum and concentric rings of decreasing strength (Fig. 3). This same pattern describes the response of a circular antenna to plane waves coming from different angles, and it is then called an antenna beam pattern.

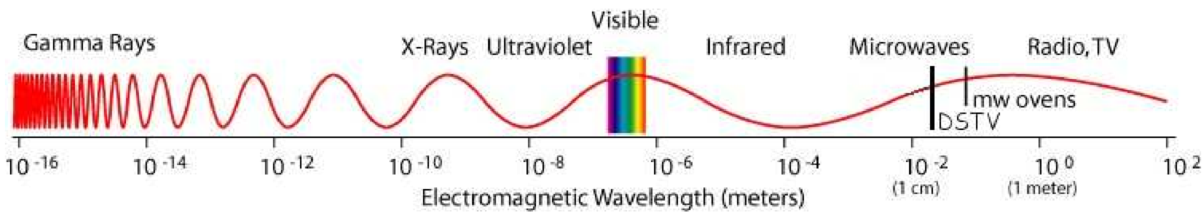

Figure 1: *The electromagnetic spectrum.*

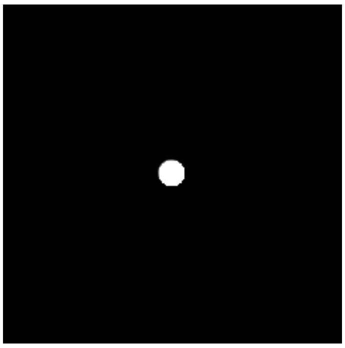

Figure 2: *A focusing lens or reflector with a circular aperture.*

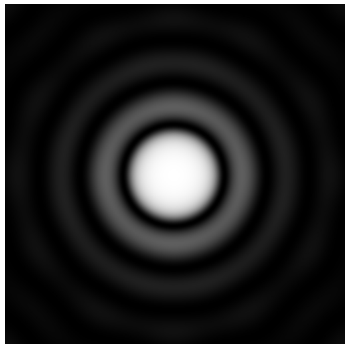

Figure 3: *The diffraction pattern produced by a circular focusing lens or reflector.*

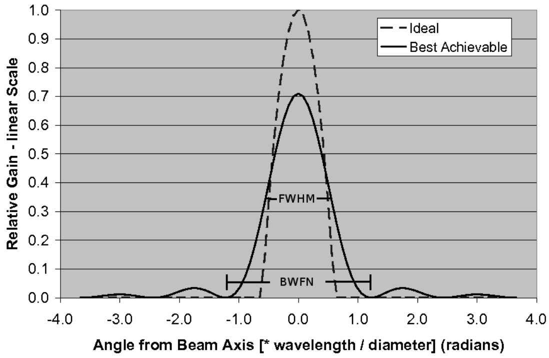

Figure 4: *Cross-sections through the beam pattern of an ideal antenna and a practically realizable antenna, shown with a linear vertical scale.*

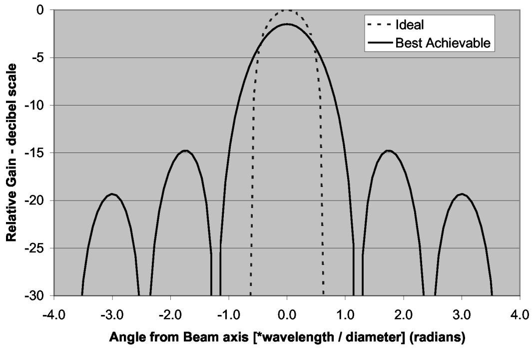

Figure 5: *Beam cross-sections of ideal and practically realizable antennas, with a logarithmic vertical scale to show the sidelobe structure.*

An "ideal" antenna would produce a beam that captures 100% of the incoming energy in the main beam and would have no sidelobes. This antenna would have a "main beam efficiency" ϵ_m of 1.0. It is not possible to actually achieve this, and ϵ_m usually lies between 0.6 and 0.8.

Figs. 4 and 5 show an ideal and an actual beam pattern in cross-section on linear and logarithmic scales. The "ideal" pattern has been modelled here with a parabolic shape, while the mathematical form of the real pattern is a $(\sin X/X)^2$ function. This describes the diffraction pattern where nothing obstructs the path of the waves, i.e. it has an "unblocked aperture", and is uniformly illuminated. The first minimum or null in the pattern occurs at a radius of about $1.2\lambda/D$ radians, so the beamwidth to first nulls is

$$BWFN \sim 2.4\lambda/D \quad \text{[radians]}. \tag{1}$$

The beamwidth at the half-power points (HPBW), also called the Full Width at Half Maximum (FWHM), is about half this, as shown in figs. 4 and 5.

$$HPBW = FWHM \sim BWFN/2 = 1.2\lambda/D \quad \text{[radians]}. \tag{2}$$

1.3 Brightness Temperature and Antenna Temperature

Fig. 6 shows the brightness as a function of frequency for several black body radiators modelled as having equal size but different temperatures. The frequencies of satellite TV transmission and visible light are marked. Clearly, hotter objects produce more radiation than cooler ones, and the brightness maximum occurs at a higher frequency. The wavelength or frequency at which the intensity peaks is given by the well-known Wien displacement law.

From Fig. 6 we can see that for all objects with temperatures more than a few degrees above absolute zero, the brightness peak occurs well above the operating range of radio telescopes. Hence radio telescopes work

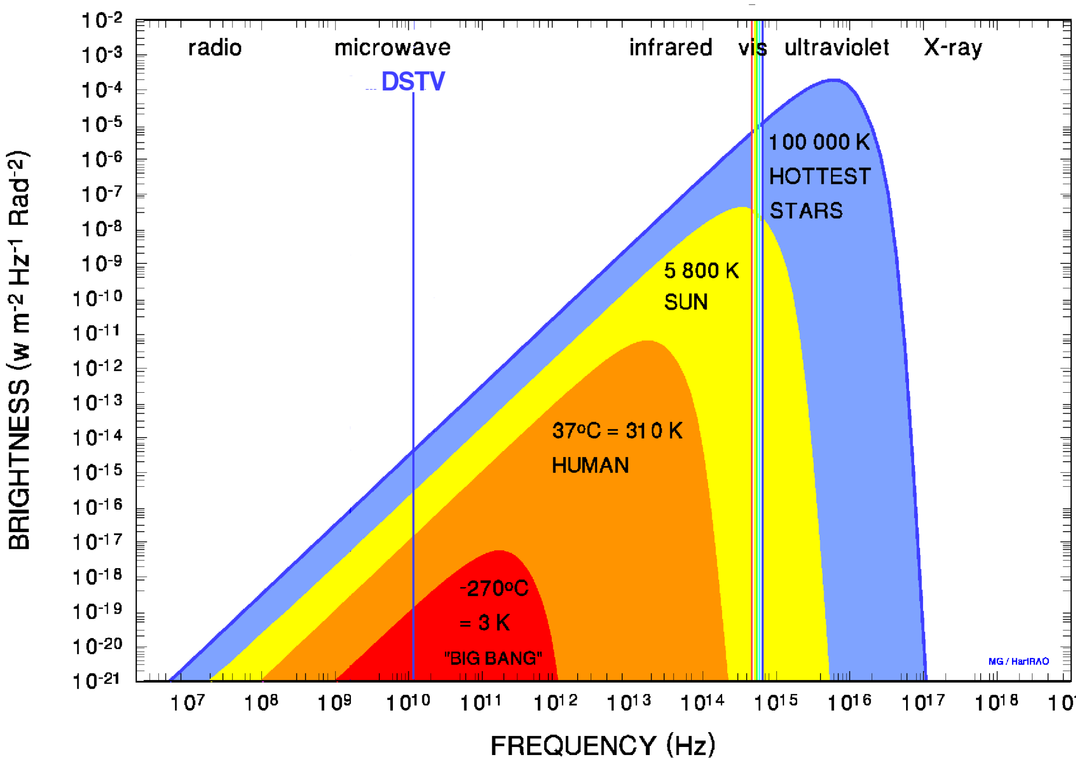

Figure 6: *Blackbody radiation from solid objects of the same angular size, at different temperatures*

in the range where $h\nu << kT$, so the Rayleigh-Jeans law applies and the brightness B - and hence the power measured by a radio telescope - is proportional to the temperature T of the emitting source:

$$B = \frac{2kT}{\lambda^2} \quad [\mathrm{W\ m^{-2}\ Hz^{-1}\ sr^{-1}}] \tag{3}$$

where k = Boltzmann's constant = 1.38×10^{-23} [J K^{-1}].

The apparent temperature of an emitting source at a given frequency is a property of the object emitting the radio waves. This is called its "**brightness temperature**", T_B.

For some astronomical objects the brightness temperature that is measured using a radio telescope is meaningful as a physical temperature, for example when observing a planet or moon. For other objects it may not be, depending on the mechanism that produced the radio emission.

By pointing the antenna at objects of known temperature that completely fill the beam, we can calibrate the output signal in units of absolute temperature (Kelvins). So one can think of a radio telescope as a remote-sensing thermometer.

The "**antenna temperature**" T_A of a source is the increase in temperature (receiver output) measured when the antenna is pointed at a radio-emitting source. It will be less than brightness temperature of the source if the source does not fill the whole beam of the telescope. Note that "antenna temperature" has nothing to do with the physical temperature of the antenna.

To obtain the brightness temperature T_B of the emitting source from its measured antenna temperature T_A, we have to measure the angular size of the source and of the telescope beam. The ratio of the angular size (solid angle) of the source to the angular size of the beam gives the fraction of the beam that is filled by the source.

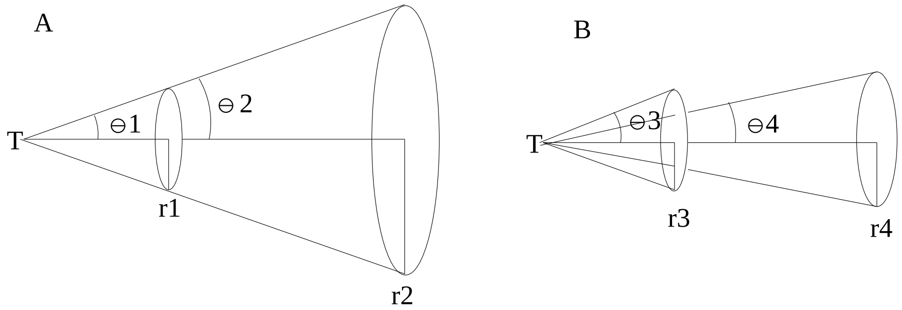

Figure 7: *Comparison of physical radius r and angular radius θ. In diagram A the two objects have different physical radii: $r2 > r1$, but the same angular radii as seen by telescope T: $\theta 1 = \theta 2$. This applies to the Moon and Sun as seen from Earth. By contrast, in diagram B, $r4 > r3$, but $\theta 4 < \theta 3$.*

1.4 Angular Sizes and the Sun's Brightness Temperature

The difference between physical diameter and angular diameter is shown in Fig. 7. For example, the Sun has a physical diameter of 1.4 million km, while the Moon has a diameter of 3500 km. Yet, as seen from the Earth, the Sun and the Moon appear to be the same size, i.e. they have the same angular diameter. How can this be? The Sun is 400 times bigger than the Moon, but it is also 400 times further away. As the projection of the antenna beam onto the sky is two-dimensional, we shall need to find the angular area that it covers. Angular area is called a "solid angle" and the units are radians^2, or steradians (sr). An object with an angular radius θ radians subtends a solid angle

$$\Omega = 2\pi(1 \; - \; cos \; \theta) \quad \text{[sr]}. \tag{4}$$

For small θ,

$$\Omega = \pi\theta^2 \quad \text{[sr]}. \tag{5}$$

We can use this equation to calculate the solid angle of the Sun as seen from the Earth, Ω_s.

The beam solid angle Ω_A of the antenna can be obtained from the half-power beamwidth (HPBW) - in units of radians - by assuming the main lobe of the beam has a Gaussian shape:

$$\Omega_A \;\sim\; 1.133\,(HPBW)^2 \quad \text{[sr]}. \tag{6}$$

The Sun's brightness temperature T_B can then be estimated by scaling its measured antenna temperature T_A by the ratio of the beam solid angle Ω_A to the Sun's solid angle Ω_s and correcting for the main beam efficiency ϵ_m being less than unity:

$$T_B = \frac{\Omega_A T_A}{\Omega_s \epsilon_m} \quad \text{[K]}. \tag{7}$$

2 Detecting Radio Emission from Space

When the telescope looks at a radio source in the sky, the receiver output is the sum of the radio waves received from several different sources:

- Behind the radio source whose brightness we want to measure is the cosmic microwave background (CMB) coming from every direction in space. This is the radiation emitted as the first atoms formed, 380 000 years after the Big Bang. The black body temperature of the CMB T_{cmb} has now decreased to 2.7 Kelvins, as the expansion of the Universe has stretched out the waves by a factor of 1000. The CMB produces a brightness temperature T_{Bcmb} of ~2.7 K at 1.4 or 4 GHz, reducing to 2.5 K at 12 GHz.
- The emission from the radio source we want to measure, which produces the antenna temperature T_A.

- Radiation from the dry atmosphere T_{at} and from the water vapour in the atmosphere T_{wv}. The dry air adds about 1K, and at 12 GHz water vapour adds 1 - 2 K, depending on the humidity.
- The radiation the feed receives through the antenna sidelobes from the (warm - ~290 K) ground or nearby buildings beyond the edge of the antenna, of brightness temperature T_g. With the antenna pointing straight up at zenith this could add 5 - 15 K; 10 K is a reasonable number to use. It increases when pointing close to the horizon.
- The amplifiers in the receiver generate their own electronic noise and so produce a receiver noise temperature T_R.

The sum of these parts is called the "system temperature" T_{sys}. Summing from the most distant noise contributor to the nearest we have:

$$T_{sys} = T_{Bcmb} + T_A + T_{at} + T_{wv} + T_g + T_R \quad \text{[K]} \tag{8}$$

3 Apparatus

3.1 A Simple Radio Telescope

The main parts of a simple radio telescope comprising a satellite dish and radiometer are shown in Fig. 8. The incoming radio waves from natural emitters are weak and noise-like. If the output of the detector is connected to a loudspeaker, the signal sounds like a hiss, as one hears if a radio is tuned off-station. The internal noise produced in the amplifiers is generally larger than the signal from natural radio sources.

3.2 Apparatus for Measuring the Antenna Beamwidth

This is optional. Apparatus required is a tripod or mount on which the satellite dish can be locked in position, a timer (e.g. watch on which seconds are displayed), pen and notepad.

3.3 Apparatus for Measuring the Diameter of the Sun

You will need two pieces of card, a pencil, a sharp knife, a ruler and a 2 metre tape measure.

4 Experimental Procedure

4.1 Calibrating the Radio Telescope

The satellite dish produces an output voltage proportional to the temperature of the object it is pointed at plus its own internal receiver temperature. We need to establish a scale of Kelvins per radiometer output unit. We do this by using the sky at zenith as a "cold load", and the ground as a "hot load". If $V1$ and $V2$ are the two meter readings and c is a constant of proportionality, then:

$$T_R + T_{sky} = cV1 \quad \text{[K]} \tag{9}$$

where $T_{sky} = T_{Bcmb} + T_{at} + T_{wv} + T_g \sim$ 15 K.

$$T_R + T_{ground} = cV2 \quad \text{[K]} \tag{10}$$

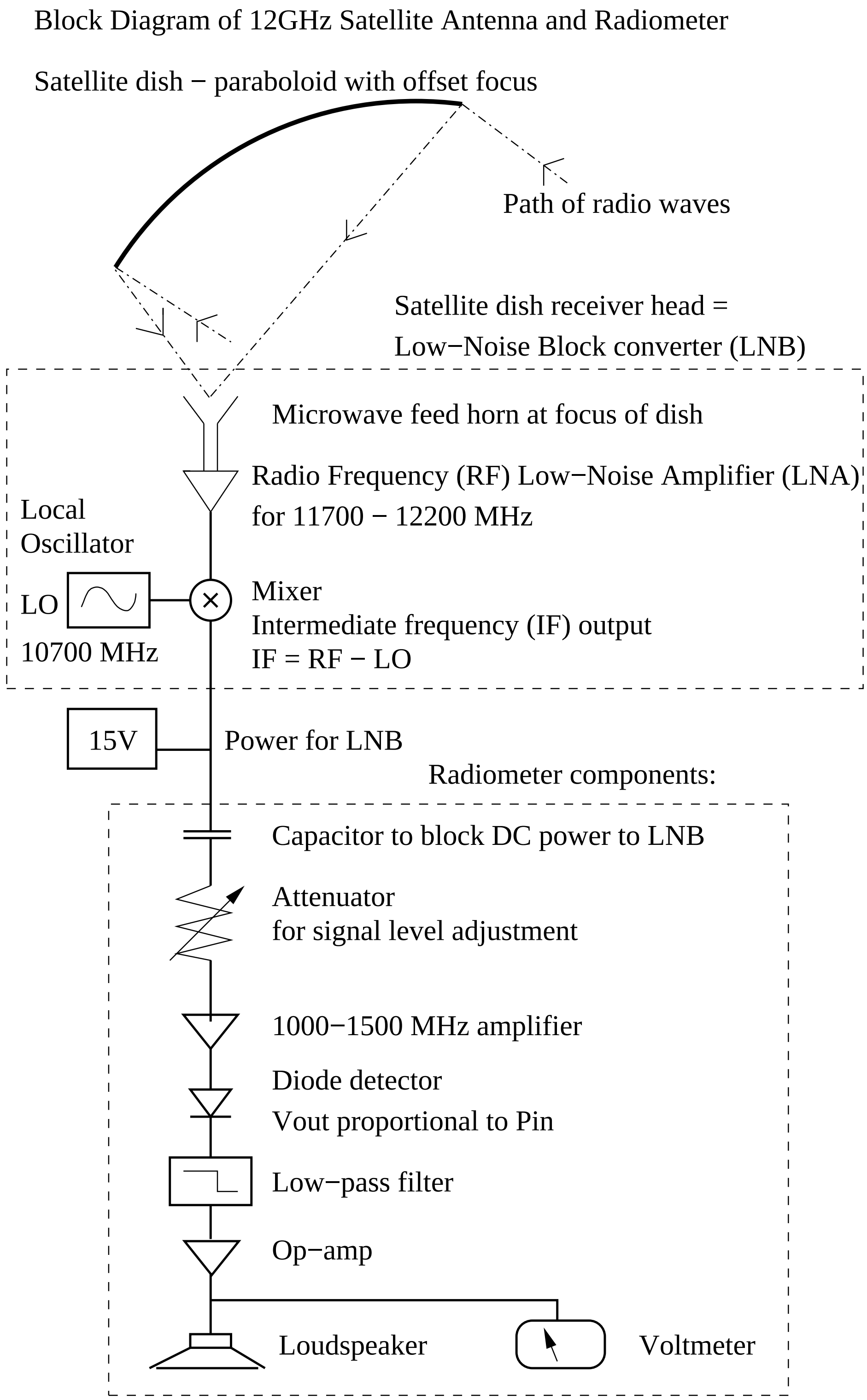

Figure 8: *Main components of a typical satellite dish and radiometer.*

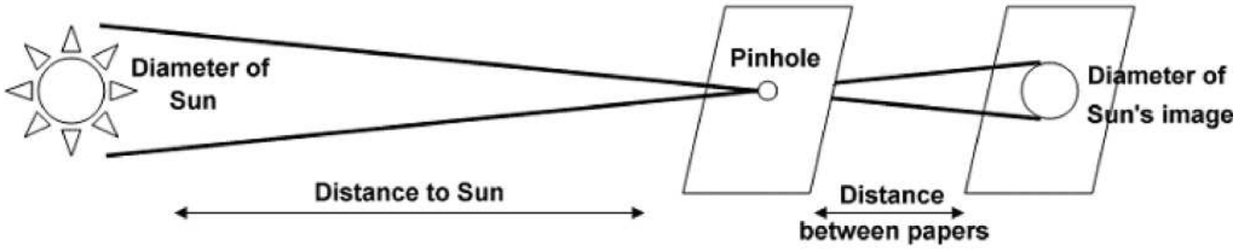

Figure 9: *Sun projection.*

T_{ground} must be measured (or estimated by touch).

Now we can use Eqns. 9 and 10 to solve for the two unknowns, c and T_R.

4.2 Measure the Antenna Temperature from the Sun

We need to aim the telescope at the Sun. With an offset-fed paraboloid this can be a litle tricky. Hold the dish horizontal. then turn the dish horizontally so the shadow of the feed arm falls across the centre of the dish. Then rotate the dish in elevation so the shadow of the feed arm on the dish gets shorter. The Sun comes into the beam just after the shadow leaves the edge of the dish. Adjust the direction gently up and down and sideways to maximise the signal from the Sun; the signal should roughly double. Take a new meter reading $V3$:

$$T_R + T_{sky} + T_{Asun} = cV3 \quad [\mathrm{K}]. \tag{11}$$

As c and T_R are known from the calibration, and we have a reasonable estimate of T_{sky}, we can immediately calculate the antenna temperature of the Sun as measured with this telescope using eqn. 11.

Next we need to measure the angular diameter of the Sun and of the telescope beam in order to calculate the solid angles they subtend.

4.3 Measure the Angular Diameter of the Sun

The usual sources of information will give the angular diameter of the Sun. However we can actually measure for ourselves the angular diameter of the photosphere - the surface of the Sun from which light is emitted.

Use pinhole projection to measure the angular diameter of the Sun. On one piece of card use a ruler and pencil to mark three equilateral triangles several cm apart, with sides of about two, three and four millimetres. Cut out the triangles using the sharp knife and ruler.

The card with the holes is used to project images of the Sun onto the second card (Fig. 9). The two cards need to be separated by a distance D, measured by tape measure. To optimise your experiment: What shape should the projected Sun have? Which hole gives the best image to work with? What distance D (large or small) will give the most accurate result? The linear diameter d of the projected Sun is measured with the ruler. The angular diameter of the Sun is d/D radians, so its angular radius Θ_s is given by:

$$\Theta_s = \frac{d}{2D} \quad [\mathrm{rad}] \tag{12}$$

From its solar radius Θ_s (Eqn. 12), calculate the solid angle subtended by the Sun Ω_s using Eqn. 5.

4.4 Measure the Half-Power Beamwidth of the Radio Telescope

The half-power beam width (HPBW) of the antenna can be estimated using Eqn. 2.

If the satellite dish is mounted on a tripod or mount so that it can be locked in position, then it is possible to carry out a "drift scan" across the Sun, as follows.

Point the antenna to get the maximum signal from the Sun. Lock the antenna's position. Immediately write down the time (minutes and seconds) and the voltage on the meter recording the signal strength, and repeat every ten seconds.

The drift scan will give a cross-section of half the antenna beam pattern, as in Fig. 4. The time for the signal to go from maximum to halfway down to minimum is equal to half of the $HPBW$, in seconds. Units of time are converted to angle by noting that the Sun moves through 1 degree in 4 minutes / cos(Sun's declination). The Sun's declination is how far north or south of the equator it is at the time of observation. Its declination is -23.44° on about Dec 21, 0° on March 21 and September 21, and +23.44° on June 21. It follows a sinusoidal path between those dates. Alternatively, get the "Apparent Dec" of the Sun for the measurement date from http://www.calsky.com/cs.cgi/Sun/1? .

This will give you the half-power beamwidth (HPBW) in degrees. Convert this to radians. Check your answer by comparing this to the estimate from Eqn. 2.

Calculate the beam solid angle Ω_A from the half-power beamwidth (estimated or measured, in radians) using Eqn. 6.

4.5 Calculate the Brightness Temperature of the Sun

To calculate the brightness temperature of the Sun we now have the Sun's solid angle Ω_s from Eqn. 5, the beam solid angle Ω_A from Eqn. 6, and the antenna temperature measured from the Sun T_A, from Eqn. 11.

We can now calculate the brightness temperature of the Sun:

$$T_{BSun} = \frac{\Omega_A T_A}{\Omega_s \epsilon_m} \quad [\mathrm{K}]. \tag{13}$$

What we are missing from Eqn. 13 is a value for the main beam efficiency ϵ_m. Experiments indicate that a reasonable value for ϵ_m for a DSTV dish at 12 GHz is about 0.75. For mesh surface satellite dishes working at 3.8 GHz, a lower value is more likely as the surface shape is less accurate, and ϵ_m is about 0.5. For a mesh dish with a 1.4GHz can feed, ϵ_m is about 0.5. These values are based on measurements with domestic satellite dishes.

Estimate the uncertainty in your result in the usual way, by propagating estimates of the error in each of your measurements. Assume an uncertainty in ϵ_m of 10%.

How does your result for the Sun's brightness temperature compare to the temperature usually quoted for the Sun's photosphere (light emitting surface)? What do you think this implies?

References

For an easy to read introduction to radio astronomy, get this free download:

Miller, D F, 1998, *Basics of Radio Astronomy for the Goldstone-Apple Valley Radio Telescope*, Jet Propulsion laboratory JPL D-13835, from http://www2.jpl.nasa.gov/radioastronomy/